Homework 0

Due: Monday, Sept. 10, 2018 at 5:00 PM

Please pay attention to the following message as there are some deviations from the usual rules.

Firstly, we are using anonymous grading, **so please write your Banner ID number at the top of your homework and not your name or login.**

This homework must be typed in LaTeX. However, we suggest drawing diagrams by hand. We recommend typing up Latex documents via `v2.overleaf.com`, which will allow you to easily collaborate with your partner on future problem sets. Also see the links at `http://cs.brown.edu/courses/cs157/content/docs/CS157LatexLinks.pdf` for more information on getting started in LaTeX.

All homework will be handed in electronically via Gradescope. We will email Gradescope instructions to the class list. Be sure to have the class in your shopping cart to receive course emails, or email the HTAs (cs1570headtas@lists.brown.edu) to be added to our email list.

For this assignment, the homework must be **written alone, *not* in pairs**. However, as usual, we encourage you to come to office hours, discuss the problems with other students, or consult online resources. **Read the collaboration policy now, if you have not already.**

Please ensure that your solutions are complete, concise, and communicated clearly: use full sentences and plan your presentation before you write. Except in the rare cases where it is indicated otherwise, consider every problem as asking you to prove your result. Writing formally about computer science concepts may be new to you—if you want advice on how to communicate your solutions (or, in fact, whether what you have constitutes a solution), **please come to TA collaboration hours!**

Do all problems for both the 1-credit and 1.5-credit tracks.

Problem 1

(5 points)

Before you start putting a lot of your time into CS157, it is important to try to figure out what you (yes, you!) will get out of the course. Think about what you might learn here that you might value 10 years from now in your job—what topics, skills, etc. might stick with you? Write *at least 3 sentences* describing how you expect to benefit from CS157. Whatever you write here, please keep it in mind through the rest of this course, as it can motivate and direct your studies.

Problem 2

(10 points)

Turn in a signed collaboration policy.

Problem 3

(20 points)

Sort the following functions by order of growth from slowest to fastest ("big-O" notation). For each pair of adjacent functions in your list, please write one sentence describing (informally is fine) why you ordered them as you did.

(a) $7n^3 - 10n$	(b) $4n^2$	(c) n
(d) $n^{8621909}$	(e) 3^n	(f) $e^{\log \log n}$
(g) $n^{\log n}$	(h) $6n \log n$	(i) $n!$

Problem 4

(30 points)

1. Given the recurrence $T_1(n) = 2 \cdot T_1(n-1) + 1$ where $T_1(0) = 0$, carefully pick (yes. actually pick.) two positive integers a and b and prove by induction that $T_1(n) \leq a \cdot 2^n - b$. (Be sure to state your induction hypothesis clearly!) Then conclude that $T_1(n) = O(2^n)$.

2. Prove by induction that $T_2(n) \leq n \log_2 n$, given that: $T_2(n) = \begin{cases} 2 \cdot T_2\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n & \text{if } n \geq 2 \\ 0 & \text{if } n = 1 \end{cases}$

   where the notation $\left\lfloor \frac{n}{2} \right\rfloor$ means "round $\frac{n}{2}$ down to the nearest integer."

Problem 5

Linear algebra review

(15 points)

This question and the next one are warmups for material we will see in class in a few weeks. Make sure you have an intuitive understanding of what these concepts mean, instead of relying on computer packages to just compute numbers.

1. (0 points, do not turn in) Remind yourself how to add matrices, multiply matrices, transpose matrices, and what a matrix inverse does. Check reference material as needed (e.g. via Google and/or Wikipedia).

2. (5 points) Express the vector $\begin{bmatrix} 7 & 0 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 3 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \end{bmatrix}$.

3. (5 points) Explain why the task in part (2) is possible in a sentence using the technical term "basis." **Important:** if you are not familiar, look up "basis" on Wikipedia for example, or ask a friend or a TA.

4. (5 points) Describe part (2) in a sentence involving the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$. Again, if you cannot think of an appropriate sentence, talk to a TA to discuss what matrix inversion means.

Problem 6

(20 points)

In this problem there is no need to rigorously prove your statements, but remember to use full sentences, and describe anything that will not be obvious to the reader. All of your answers should involve exact expressions, not decimals.

1. Plot the number $1+\sqrt{3}\,i$ on the complex plane. Express $1+\sqrt{3}\,i$ in polar coordinates: label the angle and magnitude on your plot.
2. Find the complex conjugate of $3+2i$.
3. Find the reciprocal of $3+2i$. Plot $3+2i$, its reciprocal, and its complex conjugate on the same plot.
4. Given two complex numbers in polar coordinates, how do you multiply them? Complex numbers are often expressed in polar coordinates with the notation "$r\angle\theta$" to denote the number with radius r and angle θ. We are asking you to fill in the blanks in the following expression: $(a\angle b)\cdot(c\angle d) = (\quad)\angle(\quad)$
5. Look up "Euler's Formula," and explain in a sentence or two how, for a complex number $z = x + yi$, to compute e^z in polar coordinates.
6. Given an integer k and a complex number with angle θ and magnitude r, what is the expression for the k-th power of the complex number?
7. What are *all* the complex numbers whose 6th powers are 1, expressed via their real and imaginary parts? **Hint:** use the previous part for intuition about powers of complex numbers; then part 1 will give you a hint about what these numbers will look like. There are 6 answers.