Introduction to Computer Vision

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Sept 2009

Lecture 9:

Image gradients, feature detection, correlation

Goals

- Image gradient
- Filtering as feature detection
- Convolution vs correlation
- Time permitting: images as vectors

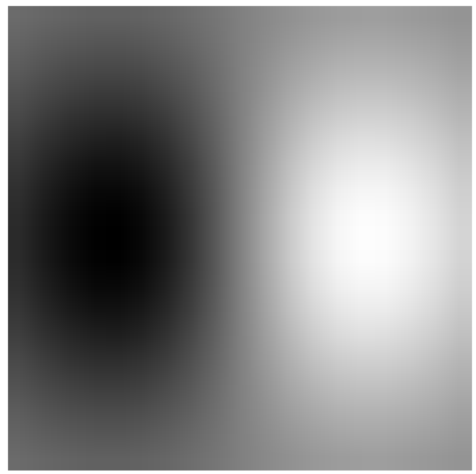
Next week

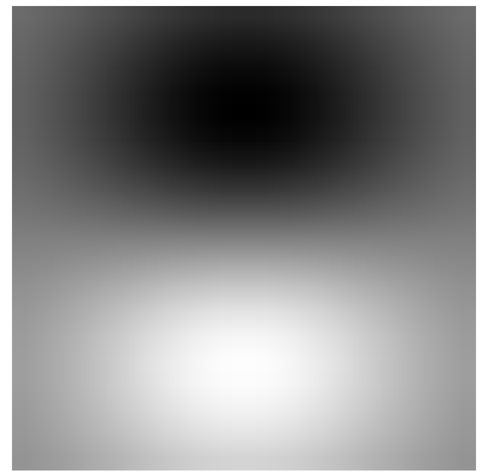
- Wednesday: data for assignment 2. important that you attend.

- Friday: Silvia Zuffi – color

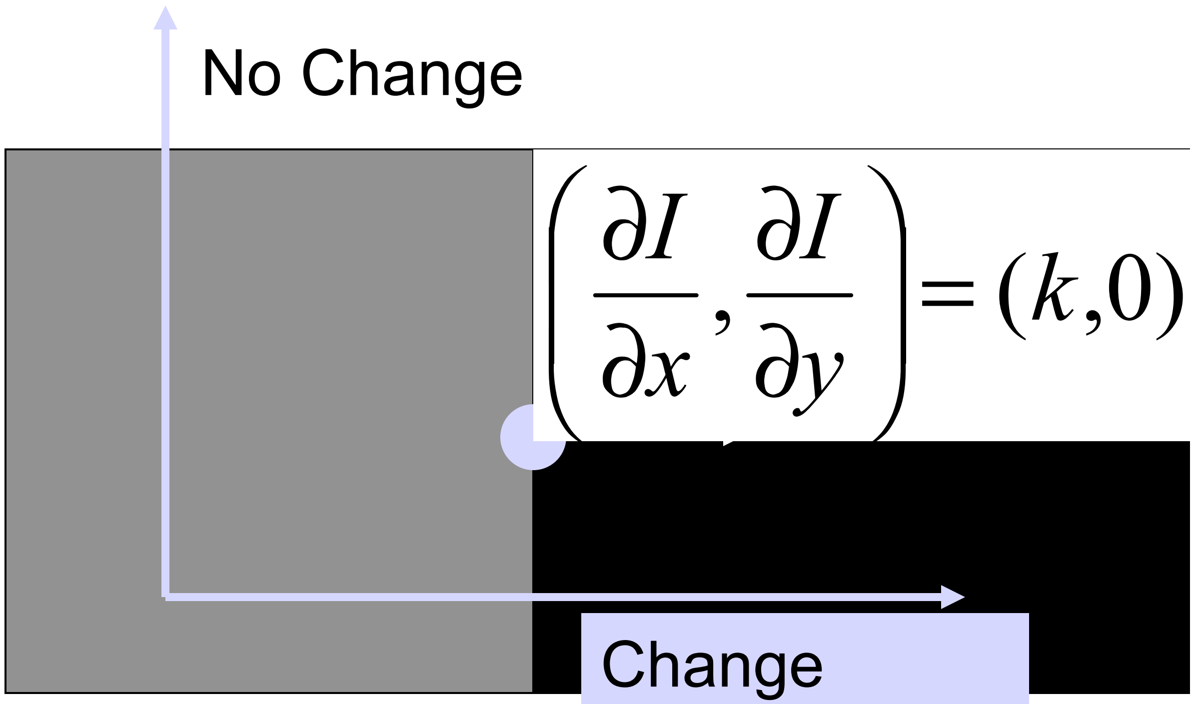
Recall derivatives of Gaussian

$$D_x \otimes (G \otimes I) = (D_x \otimes G) \otimes I$$





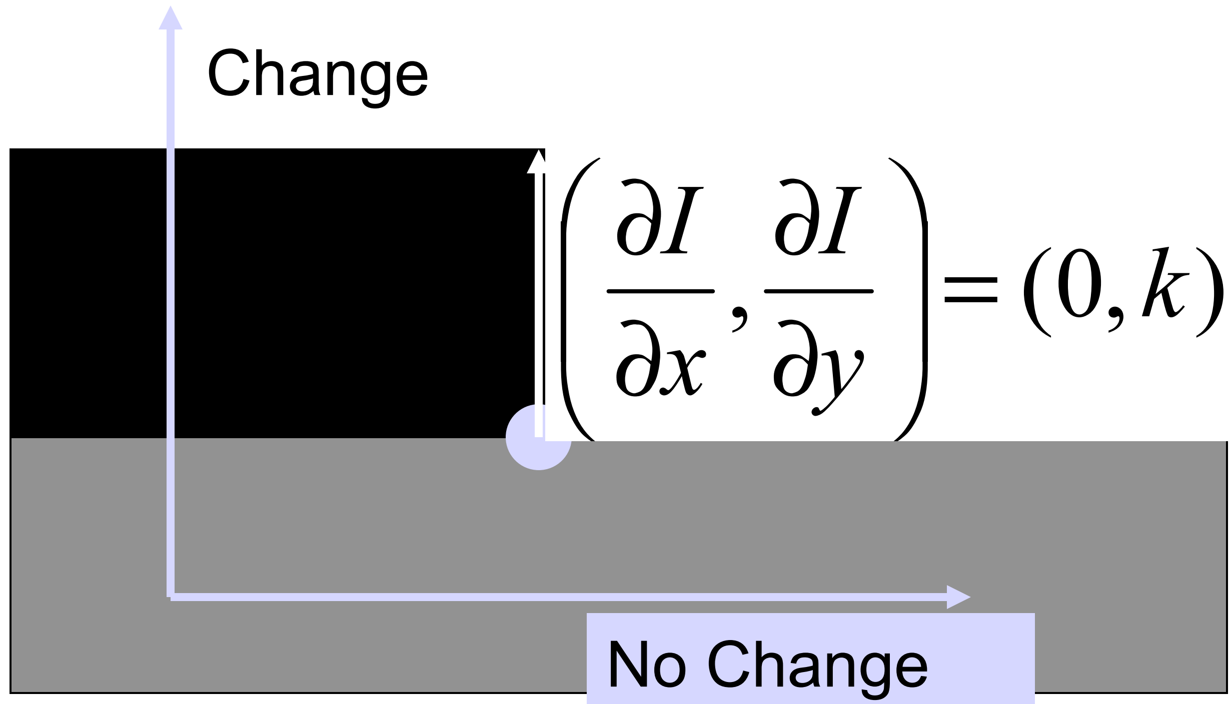
What is the gradient?



$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) = (k, 0)$$

Jacobs

What is the gradient?



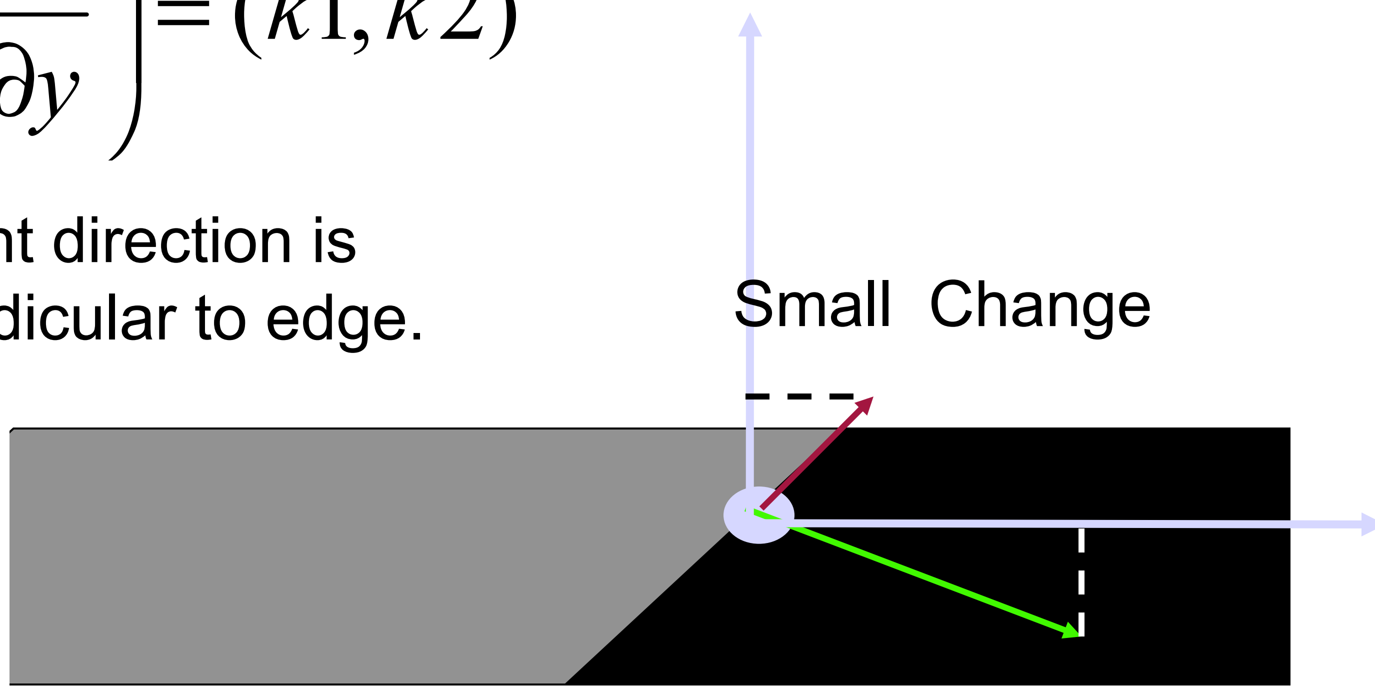
$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) = (0, k)$$

Jacobs

What is the gradient?

$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) = (k1, k2)$$

Gradient direction is perpendicular to edge.



Gradient Magnitude measures edge strength.

Jacobs

2D Edge Detection

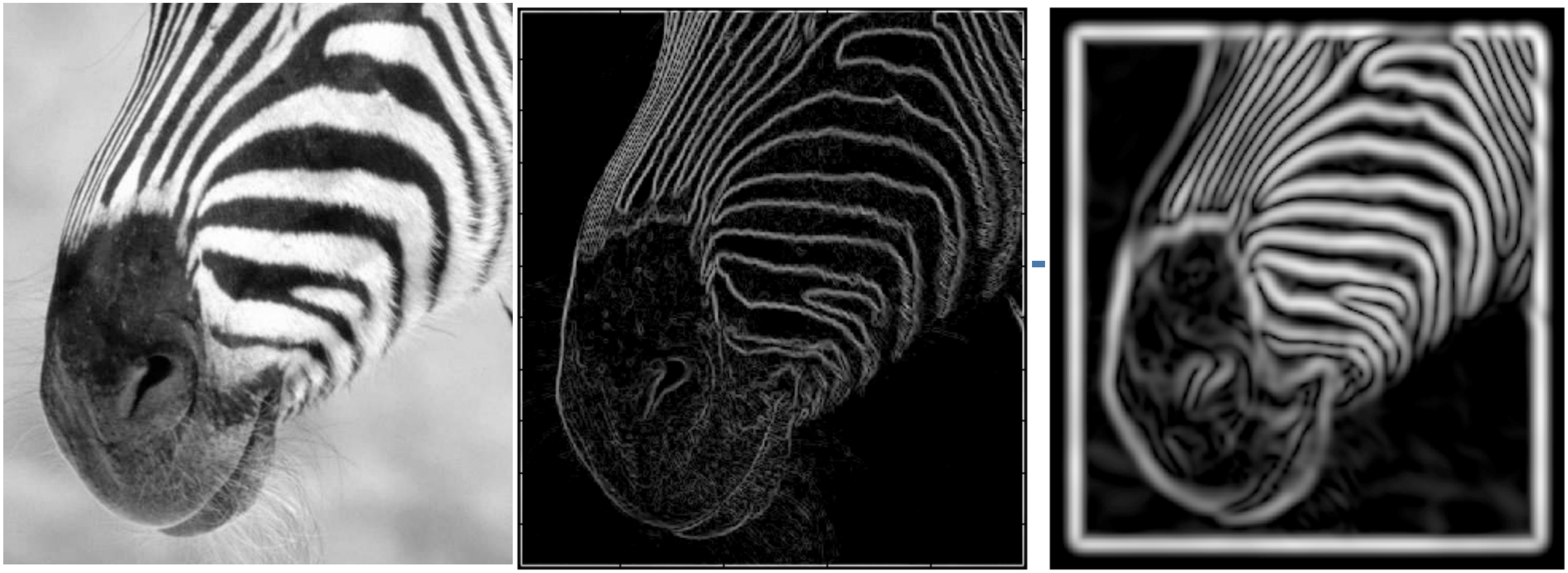
Take a derivative

– Compute the magnitude of the gradient:

$$\nabla I = (I_x, I_y) = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \text{is the Gradient}$$

$$\|\nabla I\| = \sqrt{I_x^2 + I_y^2}$$

Jacobs

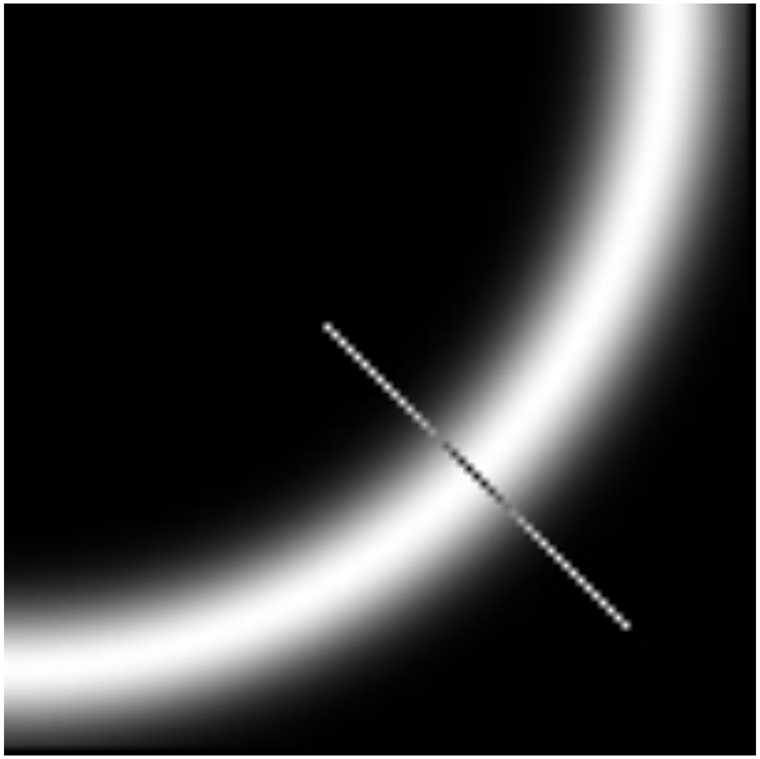


There are three major issues:

1) The gradient magnitude at different scales is different; which should we choose?
2) The gradient magnitude is large along a thick trail; how do we identify the significant points?
3) How do we link the relevant points up into curves?

Ponce & Forsyth

Non-Maxima Suppression

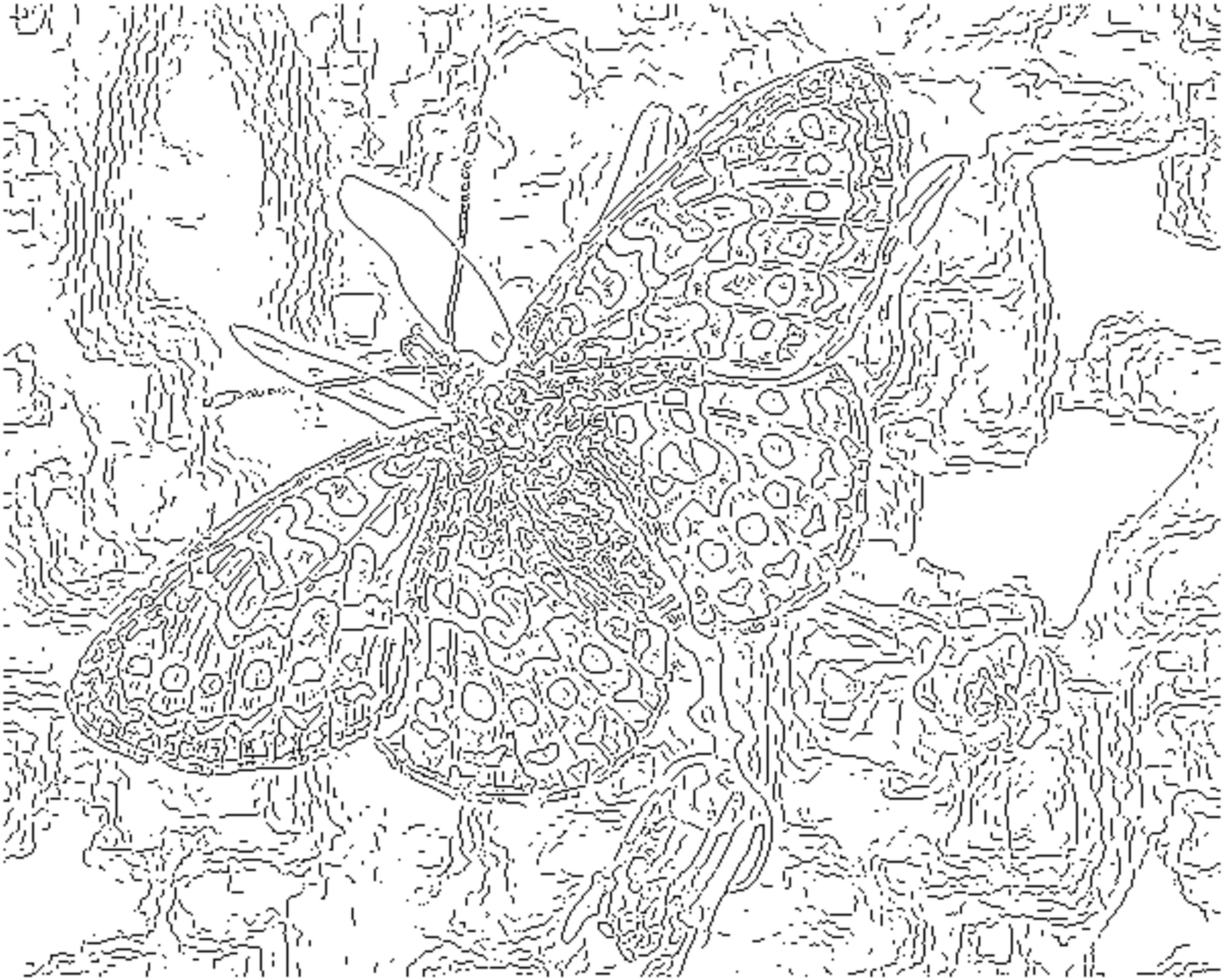


Look in a neighborhood along the direction of the gradient.

Choose the largest gradient magnitude in this neighborhood..

Ponce & Forsyth

Ponce & Forsyth



fine scale
high
threshold

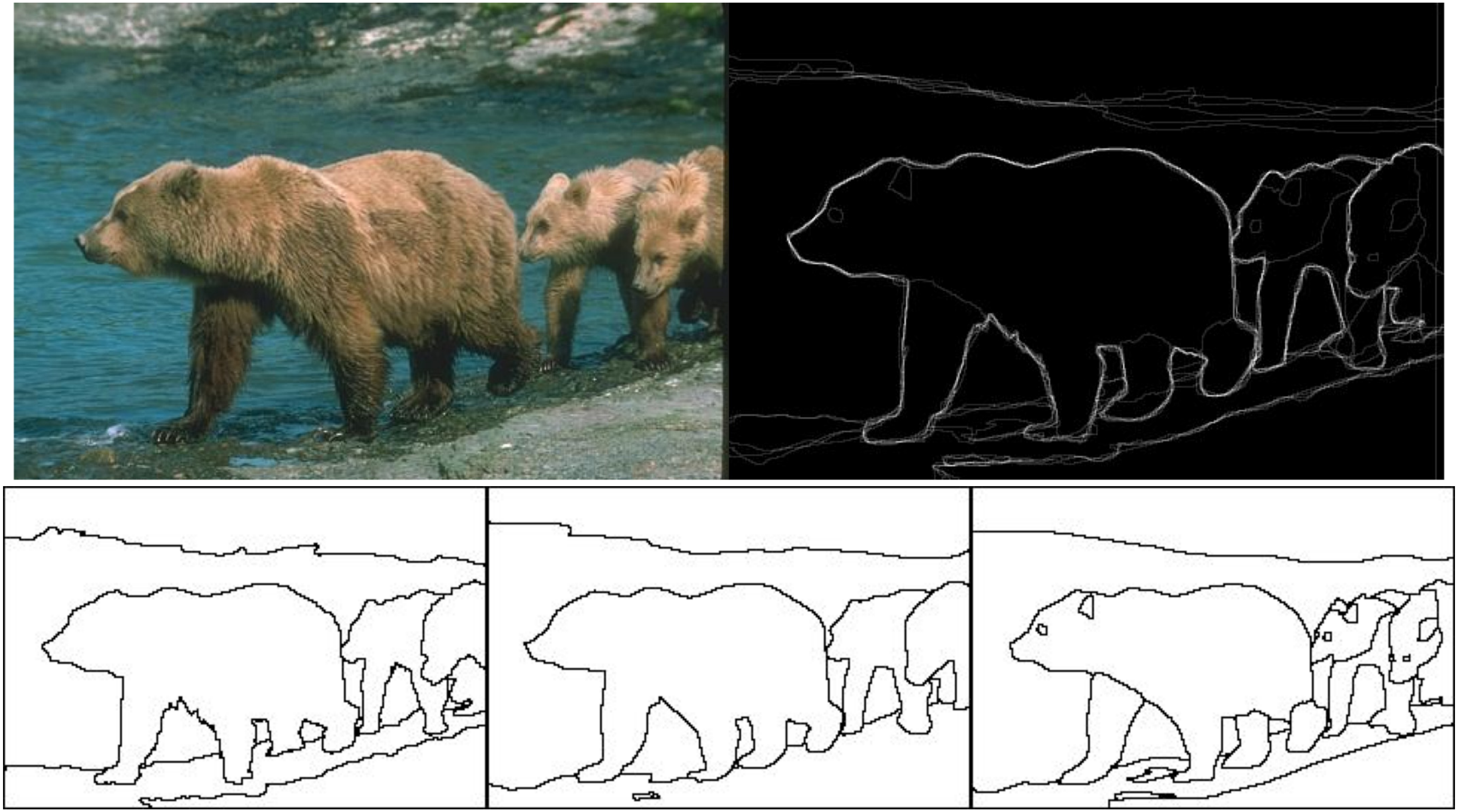
Ponce & Forsyth

coarse
scale,
high
threshold

Ponce & Forsyth

coarse
scale
low
threshold

Ponce & Forsyth

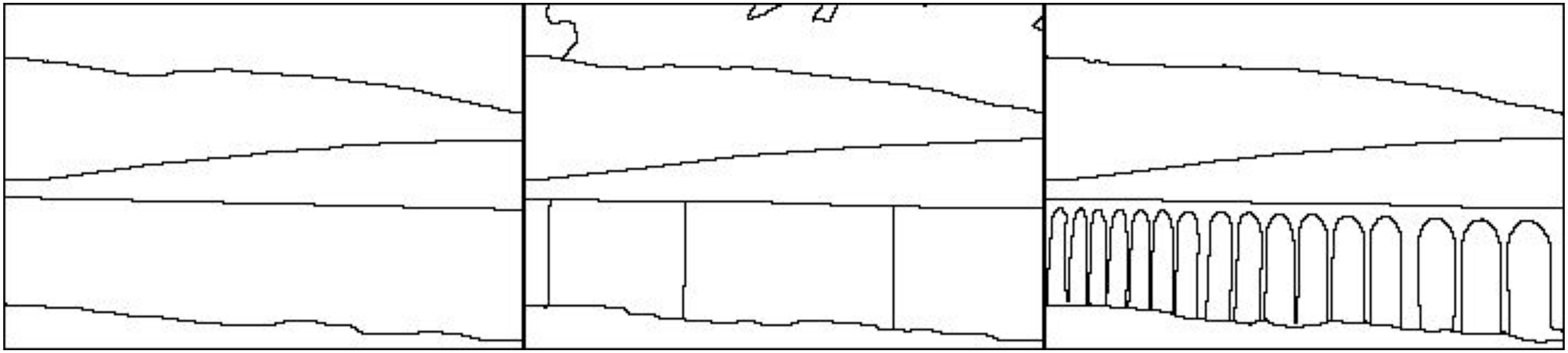


Compare these detected edges to human marked edges:

Humans focus on semantic edges and they don't always agree.

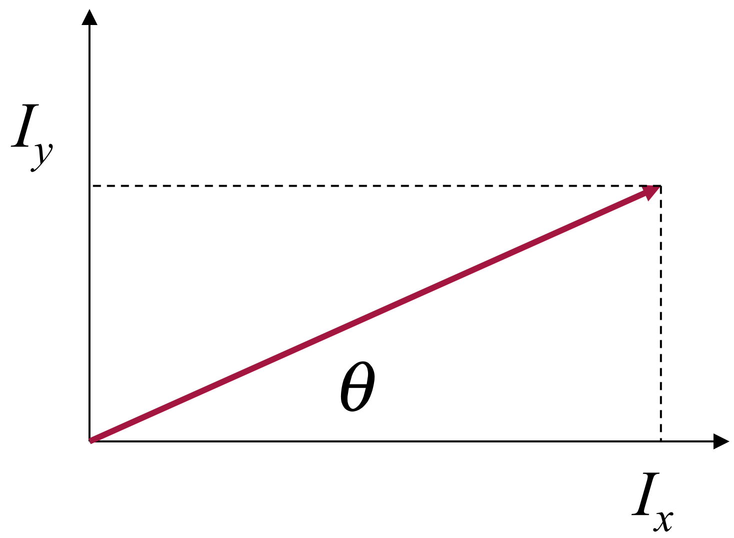
Berkeley Segmentation Dataset and Benchmark

http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/



Berkeley Segmentation Dataset and Benchmark
http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

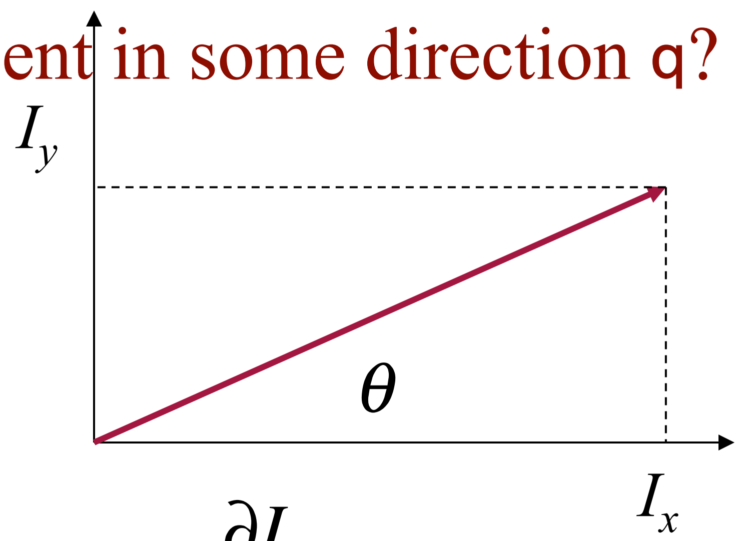
Direction of the Gradient



$$\theta(x, y) = \arctan(I_y(x, y), I_x(x, y))$$

Steerable filters

What is the gradient in some direction q?



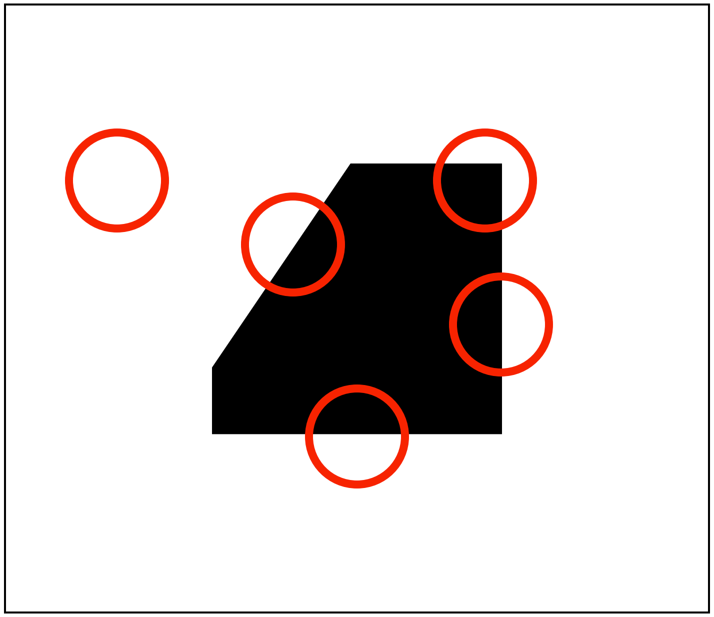
$$I_\theta(x,y) = \frac{\partial I}{\partial x}\cos\theta + \frac{\partial I}{\partial y}\sin\theta = I_x\cos\theta + I_y\sin\theta$$

In the direction of the gradient:

$$\|\nabla I\| = I_x \frac{I_x}{\|\nabla I\|} + I_y \frac{I_y}{\|\nabla I\|} = \frac{I_x^2 + I_y^2}{\sqrt{I_x^2 + I_y^2}} = \sqrt{I_x^2 + I_y^2}$$

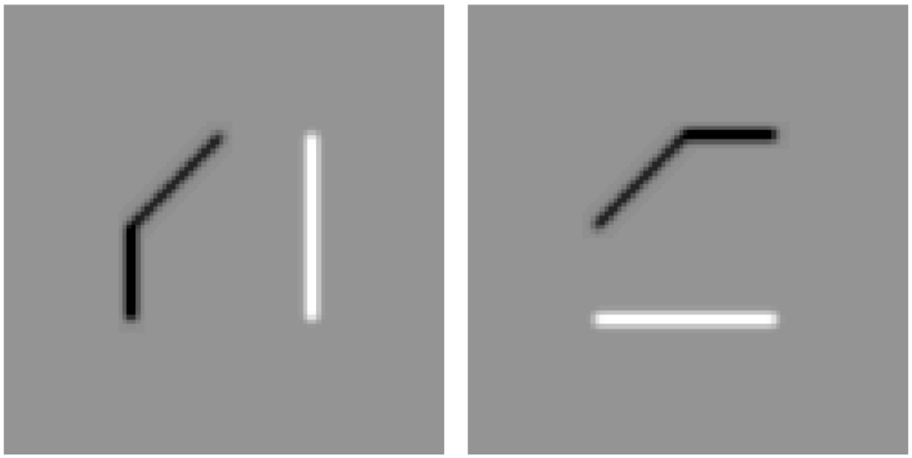
Features (problem 3)

What do the derivatives look like in these neighborhoods?

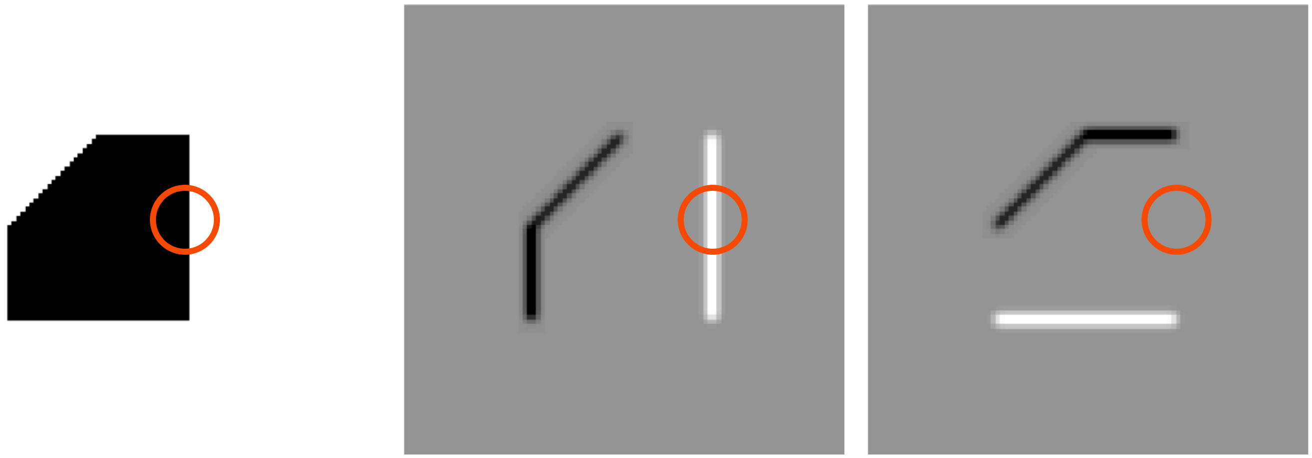


What can you tell about an image neighborhood from the local image derivatives?

Partial derivatives

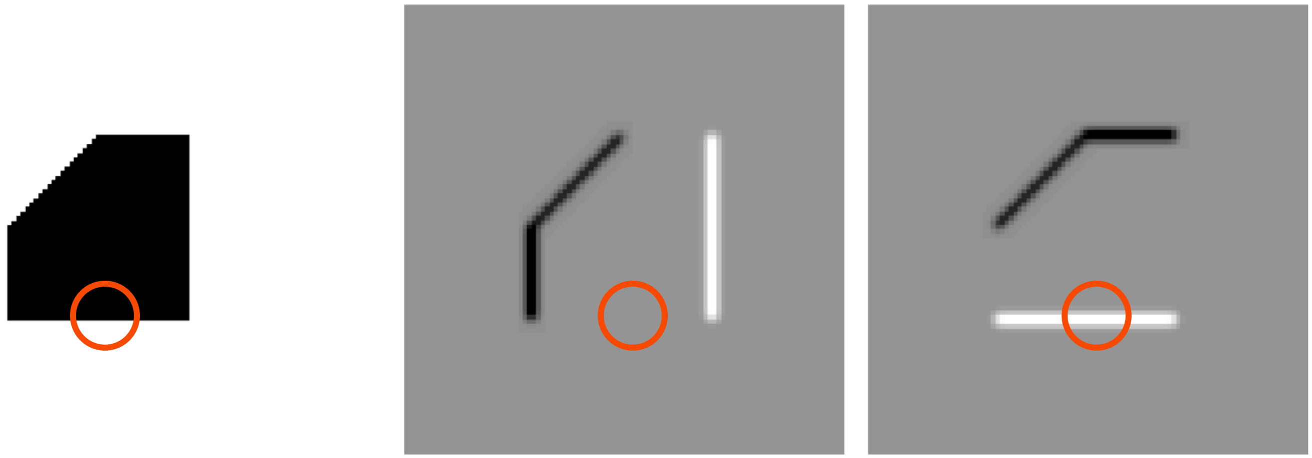


Partial derivatives



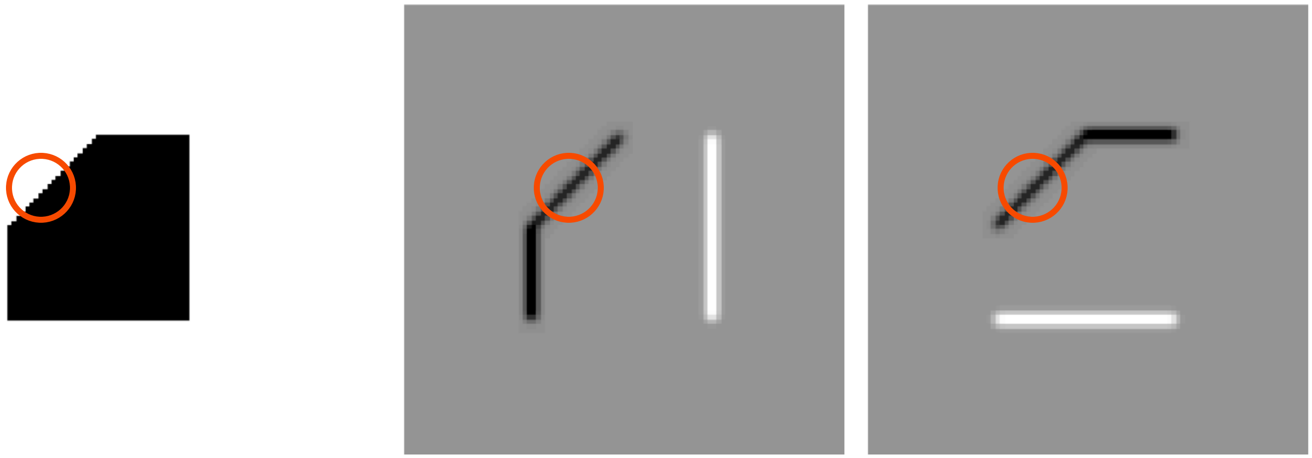
$$\begin{bmatrix} I_x & 0 & I_x & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix}$$

Partial derivatives



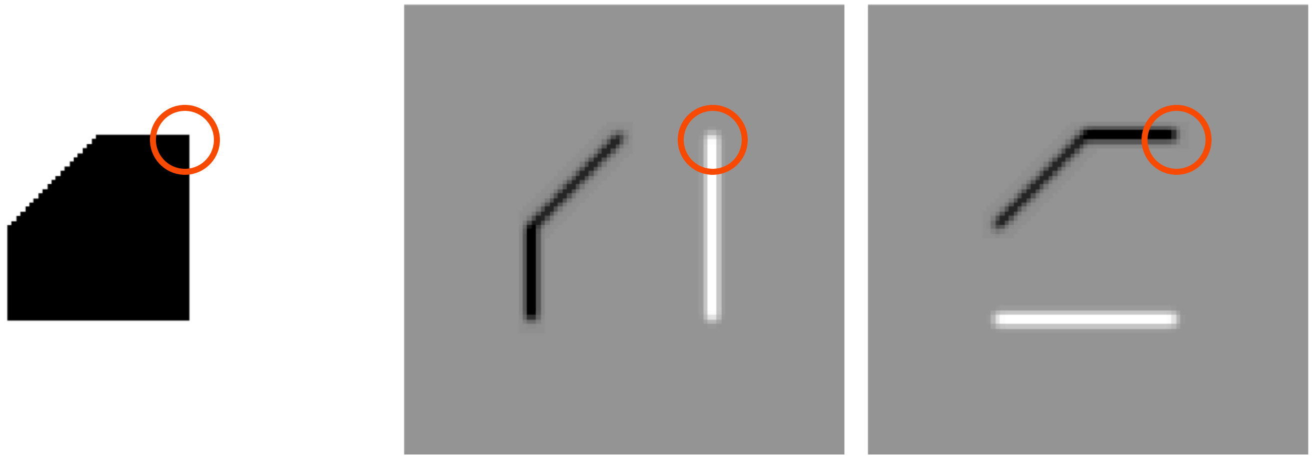
$$\begin{bmatrix} 0 & 0 & 0 & \cdots \\ I_y & I_y & 0 & \cdots \end{bmatrix}$$

Partial derivatives



$$\begin{bmatrix} 0 & I_x & I_x & \cdots \\ 0 & I_y & I_y & \cdots \end{bmatrix}$$

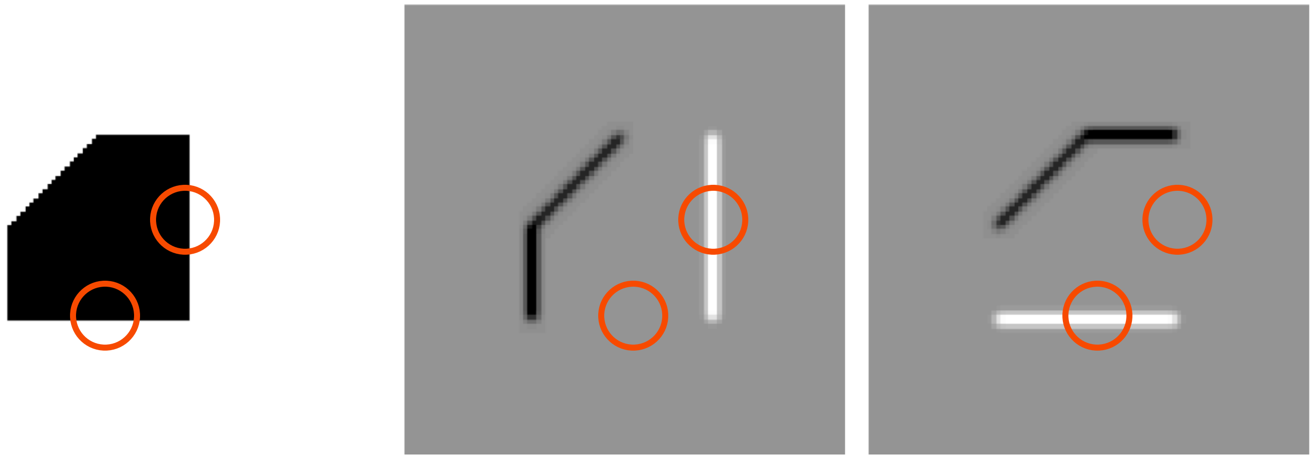
Partial derivatives



$$\begin{bmatrix} I_x & 0 & I_x & \cdots \\ 0 & I_y & I_y & \cdots \end{bmatrix}$$

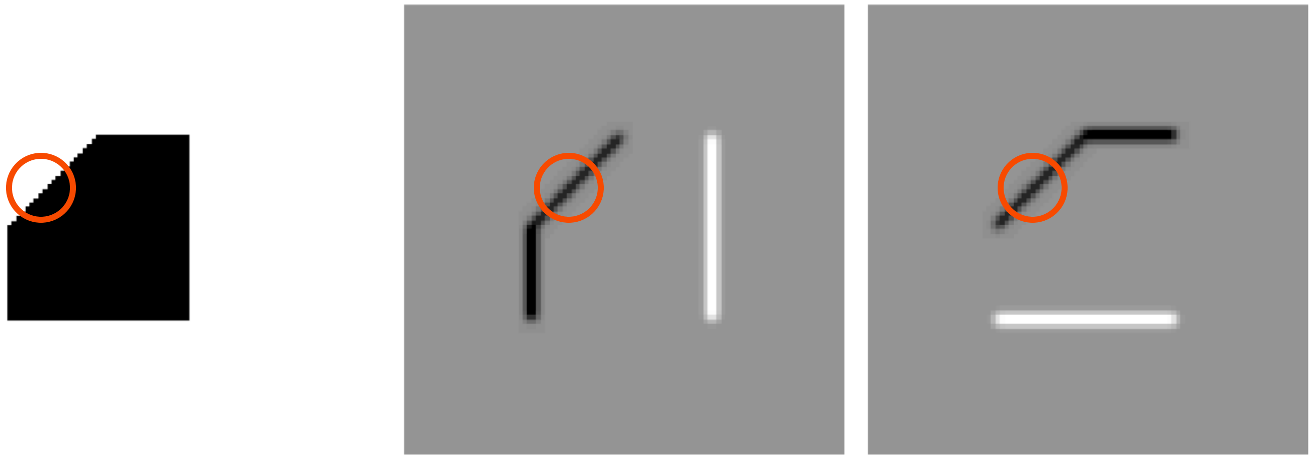
Rank of these matrices?

(ie maximum number of linearly independent columns)



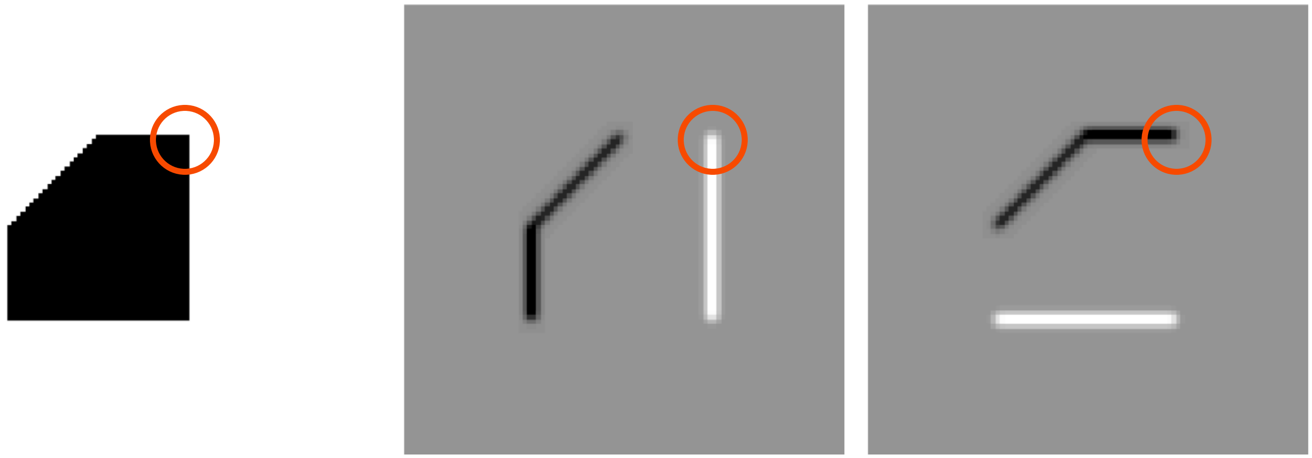
$$\begin{bmatrix} I_x & 0 & I_x & \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & \cdots \\ I_y & I_y & 0 & \cdots \end{bmatrix}$$

Rank?



$$\begin{bmatrix} 0 & I_x & I_x & \cdots \\ 0 & I_y & I_y & \cdots \end{bmatrix}$$

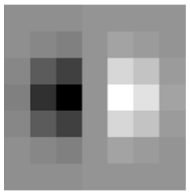
Rank?



$$\begin{bmatrix} I_x & 0 & I_x & \cdots \\ 0 & I_y & I_y & \cdots \end{bmatrix}$$

Let’s step back a moment…

Convolution   Correlation   Feature detection



f

I

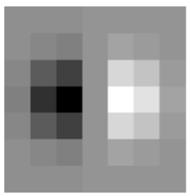
H

$$H[m,n] = f \otimes I = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

Notice the “flipping” of the filter.

Let’s step back a moment…

Convolution   Correlation   Feature detection



f

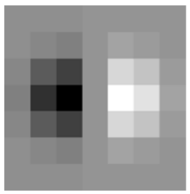
I

H

H=imfilter(I, f, 'symmetric', 'conv');

Let's step back a moment…

Convolution | **Correlation** | Feature detection



f

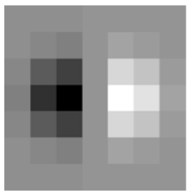
I

H

$$H[m,n] = f * I = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Let’s step back a moment…

Convolution   Correlation   Feature detection



f

I

H

dBarb=imfilter(im, dGx, 'symmetric', 'corr');

What's the difference?

Convolution:

$$H[m,n] = f \otimes I = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

Correlation:

$$H[m,n] = f * I = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Convolution is associative:

$$F \otimes (G \otimes I) = (F \otimes G) \otimes I$$

Correlation is not.

For symmetric filters, there is no difference.

Example: Correlation

[-1 3 -3 1]

[1 -2 1]

[-1 1] [1 -1] [-1 1]

[1 -2 1]

[1 -3 3 -1]

Example: Convolution

[1 -3 3 -1]

[-1 2 -1]

[-1 1] [1 -1] [-1 1]

[-1 2 -1]

[1 -3 3 -1]

Strange, eh?

- In the Fourier domain this is easy to explain (convolution is multiplication in the Fourier domain and is hence associative but correlation involves taking the complex conjugate of the filter – if the order is reversed, you take the complex conjugate of the image which changes the result).
- For this class this can essentially be ignored. From now on we'll mostly use correlation.
- If you don't believe it, try out the Matlab script on the web for this lecture.

Convolution vs Correlation

$$x' = u - x, \; dx' = -dx$$

$$\mathrm{C}(u) = \int_{-\infty}^{\infty} \mathrm{f}(x)\,\mathrm{g}(u - x)\,dx$$

$$= -\int_{\infty}^{-\infty} \mathrm{f}(u - x')\,\mathrm{g}(x')\,dx'$$

$$= \int_{-\infty}^{\infty} \mathrm{g}(x')\,\mathrm{f}(u - x')\,dx'$$

$$= \int_{-\infty}^{\infty} \mathrm{g}(x)\,\mathrm{f}(u - x)\,dx$$

substitute $x' = x + u$

$$\mathrm{f} \circ \mathrm{g} = \int_{-\infty}^{\infty} \mathrm{f}(x)\,\mathrm{g}(x + u)\,dx;$$

$$= \int_{-\infty}^{\infty} \mathrm{f}(x' - u)\,\mathrm{g}(x')\,dx'$$

$$= \int_{-\infty}^{\infty} \mathrm{f}(x - u)\,\mathrm{g}(x)\,dx$$

http://www-structmed.cimr.cam.ac.uk/Course/Convolution/convolution.html