Algorithm 2 Shapes

Introduction to Computer Graphics, Fall 2015

Solution Key

Instructions: Write or type all of your answers on a separate sheet of paper and hand in to the cs123 hand-in bin. Late hand-ins will receive no credit. Respect the CS123 collaboration policy. Do not ask a friend to hand in the assignment for you. Wikipedia is neither a trusted nor an authorized resource.

This assignment is worth 11% of your final grade for Shapes. To complete this assignment, you’ll need to look at the shapes project demo: /course/cs123/bin/cs123_demo

Notation: “p1” and “p2” refer to the values of Parameter 1 and Parameter 2, respectively.

1 Cube

[1 point] Take a look at one face of the cube. Change the tessellation parameter (p1). Notice how the number of triangles on the face changes. When p1 is n, how many triangles are on one face?

Solution: 2n^2

[1 point] Consider a unit cube at the origin with tessellation parameter = 2. Its front face lies in the +YZ plane. What are the normal vectors that correspond with each of the eight triangles that make up this face? (Note: when asked for a normal, you should always give a normalized vector, meaning a vector of length one.)

Solution: They are all (1, 0, 0).

2 Cylinder

[1½ points] The caps of the cylinder are regular polygons with N sides (an “N-gon”), where N’s value is determined by p2. You will notice they are cut up like a pizza with N slices which are isosceles triangles. The vertices of the N-gon lie on a perfect circle in the XZ plane. How will you figure out the coordinates of these vertices in terms of the radius (0.5) and the parameter θ? (What is the equation of the circle that they lie on?)

Solution: The circle lies on the planes y = ±1/2 and has the equation x^2 + z^2 = (1/2)^2. You can use the equations x = r · sinθ and z = r · cosθ to determine the x and z values of the points on the circle. Here r = 0.5; the angle θ is different for each vertex. We divide the full circle by parameter 2 to determine the spacing between each θ.

[1½ points] What is the surface normal of the point (\(\frac{\sqrt{2}}{4}, \frac{3}{4}, -\frac{\sqrt{2}}{4}\)), which lies on the barrel of the cylinder? What about an arbitrary point on the barrel of the cylinder (in terms of θ)?

Solution: (\(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\)), and (cosθ, 0, sinθ)

3 Cone

[1 point] Look at the cone with Y-axis rotation = 0 degrees, and X-axis rotation = 0 degrees. How many triangles make up one of the p2 "sides" of the cone when p1 = 1? When p1 = 3? 5? n?

Solution: You can see that when p1=1, there is just one triangle that makes up one side of the cone. As you increase p1, two triangles are added each time to the side. So there are (2 · p1) − 1 triangles on each side.

[1 point] What is the surface normal at the tip of the cone? A singularity does not have a normal. You may achieve a good shading effect by thinking of p2 vectors with their base at the tip of the cone, each pointing outward, normal from the face of the triangle associated with it. This implies that there will not be a unique normal at the tip of the cone. Each of the p2 attached triangles will have a normal dependent on its own position in the surface; specifically, the normal for a given triangle at the tip should be normal to the plane defined by that specific triangle. (Think about how OpenGL can use this information to make a realistic point at the top of the cone) Draw a simple schematic sketch illustrating the normal for all of the triangles at the tip. As long as it is clear that you "get the idea", you will receive full credit.

Solution:

Figure 1: Halo surrounding tip of cone

[1 point] Take the two dimensional line formed by the points (0, \(\frac{1}{2}\)) and (\(-\frac{1}{2}, \frac{1}{2}\)) and find its slope, m. Your answer should be a number, not in terms of m.

Solution: m = 2
Figure 2: The line representing the sloping side of a cone

[1 point] The line in Figure 2 represents the sloping side of the cone. Then, \(-\frac{1}{m}\) is the slope perpendicular to this line. Using this perpendicular slope, we can find the vertical and horizontal components of the normal on the cone body. The vertical component is the component in the XY plane. What is the magnitude of this vertical component in the normalized normal vector?

[1 point] The component in the \(x\) direction is the horizontal component. What is the magnitude of this component in the normalized normal vector?

Solution:

\[
\begin{align*}
\text{(0, 0, b)} & \quad \text{b} = \sqrt{\frac{2}{5}} \\
\text{(0, 0, c)} & \quad \text{c} = \sqrt{\frac{1}{5}} \\
\text{Then the component in the XZ plane is } & \quad \text{2b} = 2\sqrt{5} \\
\text{and the component in the y direction is } & \quad \text{b} = 1\sqrt{5}
\end{align*}
\]

4 Sphere

[1 point] In Shapes, you’ll need to compute normals yourself. What is the surface normal of the sphere at an arbitrary surface point \((r, \phi, \theta) = (0.5, \pi/4, \pi/2)\) = ? Give your answer in Cartesian coordinates.

Solution: \((0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\)

Note that the sphere in the demo is tessellated in the latitude-longitude manner, so the points you want to calculate are straight spherical coordinates. The two parameters can be used as \(\theta\) and \(\phi\), or longitude and latitude. The conversion from spherical to Cartesian coordinates is given by...

\[
\begin{align*}
x &= r \cdot \sin \phi \cdot \cos \theta \\
y &= r \cdot \cos \phi \\
z &= r \cdot \sin \phi \cdot \sin \theta
\end{align*}
\]