`11-15`

CS 53, Fall 2017

Due Nov. 17 at 2:59 pm

Problem 1 (paper hand-in): Let `vlist` consist of the following vectors with domain $D = \{a, b\}$:
`Vec(D, {a:2,b:5})`, `Vec(D, {a:8,b:10})`, `Vec(D,{a:-4,b:12})`, `Vec(D,{a:1,b:-4})`. By running `orthogonalize` on this list, we get four vectors:
`Vec(D, {a:2,b:5})`, `Vec(D, {a:3.45,b:-1.38})`, `Vec(D, {})`, `Vec(D, {})`.

We write the relationship in terms of a matrix equation:

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline a & 2 & 8 & -4 & 1 \\ b & 5 & 10 & 12 & -4 \end{array} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline a & 2 & 3.45 & 0 & 0 \\ b & 5 & -1.38 & 0 & 0 \end{array} * \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 2.28 & 1.79 & -0.621 \\ 1 & 0 & 1 & -2.2 & 0.65 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{array}$$

The columns of the first matrix are the original vectors, and the columns of the second matrix are the starred vectors.

1. Give a basis for the vector space spanned by the original vectors. Explain how you got the vectors forming the basis.
2. Use a procedure from the module `triangular` to find a basis for the null space of the matrix whose columns are the original vectors. Show your code, show the answer you get, and explain.

Problem 2: Write a procedure `basis(vlist)` with the following spec:

- *input:* a list *vlist* of Vecs
- *output:* a list of linearly independent Vecs that span the same space as *vlist*

  The Vecs returned should be elements of `orthogonalize(vlist)`.

Your procedure should use the procedure `orthogonalize` defined in the provided module `orthog` but should call no other procedures. Ideally, it should be a one-line procedure.

When given the Vecs corresponding to

$$[2, 4, 3, 5, 0], [4, -2, -5, 4, 0], [-8, 14, 21, -2, 0],$$
$$[-1, -4, -4, 0, 0], [-2, -18, -19, -6, 0], [5, -3, 1, -5, 2]$$

the procedure might return Vecs that approximately correspond to

$$[2,4,3,5,0],[3.81,-2.37,-5.28,3.54,0],$$
$$[-1.58,-0.73,0.0009,1.21,0],[0.35,-3.16,1.01,-0.99,2]$$

Note: In this problem and the next, to test whether a vector $\boldsymbol{v}$ should be considered a zero vector, you can see if the square of its norm is very small, e.g. less than 10^{-20}.

Problem 3: Write a procedure `subset_basis(vlist)` with the following spec:

- *input:* a list $vlist$ of vectors
- *output:* a list of linearly independent vectors that span the same space as $vlist$ and that are in $vlist$

Your procedure should use `orthogonalize(vlist)` and no other procedure. Ideally, it should be a one-line procedure.

When given the Vecs corresponding to

$$[2,4,3,5,0],[4,-2,-5,4,0],[-8,14,21,-2,0],$$
$$[-1,-4,-4,0,0],[-2,-18,-19,-6,0],[5,-3,1,-5,2]$$

the procedure should return the Vecs corresponding to

$$[2,4,3,5,0],[4,-2,-5,4,0],[-1,-4,-4,0,0],[5,-3,1,-5,2]$$