11-10

CS 53, Fall 2017

Due Nov. 13 at 2:59 pm

Be sure to show your work in calculations. (I don't mean show your arithmetic.) Be neat! Write legibly!

Problem 1: Define datapoints $\boldsymbol{p}_1 = [10, 30], \boldsymbol{p}_2 = [10, 10], \boldsymbol{p}_3 = [30, 20]$. Compute the sum of squared distances from the datapoints to $[20, 15]$. Then compute the centroid of the datapoints, and compute the sum of squared distances from the datapoints to the centroid.

Problem 2: Let $\boldsymbol{b} = [1, \frac{1}{2}], \boldsymbol{v}_1 = [2, 2], \boldsymbol{v}_2 = [2, 0]$.

1. Compute $\boldsymbol{b}^{||\boldsymbol{v}_1}$ and $\boldsymbol{b}^{||\boldsymbol{v}_2}$.
2. Let $\hat{\boldsymbol{b}}$ be the sum of the projections $\boldsymbol{b}^{||\boldsymbol{v}_1}$ and $\boldsymbol{b}^{||\boldsymbol{v}_2}$. We want to know if $\boldsymbol{b} - \hat{\boldsymbol{b}}$ is orthogonal to $\boldsymbol{v}_1$ Consider the following equations

$$\begin{aligned}\left\langle \boldsymbol{b} - \hat{\boldsymbol{b}}, \boldsymbol{v}_1 \right\rangle &= \langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle - \left\langle \hat{\boldsymbol{b}}, \boldsymbol{v}_1 \right\rangle \\ &= \langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_1} + \boldsymbol{b}^{||\boldsymbol{v}_2}, \boldsymbol{v}_1 \right\rangle \\ &= \langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_1}, \boldsymbol{v}_1 \right\rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_2}, \boldsymbol{v}_1 \right\rangle\end{aligned}$$

   Calculate the vaue of the final expression on the right-hand side, $\langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_1}, \boldsymbol{v}_1 \right\rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_2}, \boldsymbol{v}_1 \right\rangle$, by substituting $[1, \frac{1}{2}]$ for $\boldsymbol{b}$ and $[2, 2]$ for $\boldsymbol{v}_1$ and the vectors you computed for $\boldsymbol{b}^{||\boldsymbol{v}_1}$ and $\boldsymbol{b}^{||\boldsymbol{v}_2}$, and check if the result is zero.
3. We also want to know if $\boldsymbol{b} - \hat{\boldsymbol{b}}$ is orthogonal to $\boldsymbol{v}_2$.
   (a) Write down the above equations, suitably modified to compute the inner product of $\boldsymbol{b} - \hat{\boldsymbol{b}}$ with $\boldsymbol{v}_2$. The final expression should be

$$\langle \boldsymbol{b}, \boldsymbol{v}_2 \rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_1}, \boldsymbol{v}_2 \right\rangle - \left\langle \boldsymbol{b}^{||\boldsymbol{v}_2}, \boldsymbol{v}_2 \right\rangle$$

   (b) Perform the analogous substitutions into this expression, and calculate the value, and check if it is zero.

Problem 3: Repeat the above problem but with one change: $\boldsymbol{v}_2 = [2, -2]$.

Problem 4: See the lecture slides. I show that

$$\begin{aligned}\left\langle \boldsymbol{b} - \hat{\boldsymbol{b}}, \boldsymbol{v}_1 \right\rangle &= \langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle - \left\langle \hat{\boldsymbol{b}}, \boldsymbol{v}_1 \right\rangle \\ &= \langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle - \langle \sigma_1 \boldsymbol{v}_1 + \sigma_2 \boldsymbol{v}_2 + \cdots + \sigma_n \boldsymbol{v}_n, \boldsymbol{v}_1 \rangle\end{aligned}$$

Ignoring the cross-terms, show using algebra that $\langle \boldsymbol{b}, \boldsymbol{v}_1 \rangle = \langle \sigma \boldsymbol{v}_1, \boldsymbol{v}_1 \rangle$ by using the definition of σ_1.