09-22

CS 53, Fall 2017

Due September 25 at 2:59 pm

I have given all these problems together because they are related to each other but you are required to do only the first four problems for September 25. The remaining two problems will be part of the next homework.

I have not given you general methods to solve these problems. However, you should be able to solve them with basic algebra, once you understand the material in the lectures of Sept. 20 and Sept 22. Stay calm and think through the material.

Problem 1: Find a set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ of vectors (how many is up to you), each represented by a list, such that the solution set of

$$\begin{aligned} [1,1,1] \cdot \boldsymbol{x} &= 0 \\ [1,0,1] \cdot \boldsymbol{x} &= 0 \end{aligned}$$

is Span $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$

Problem 2: Find a list $[\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m]$ of vectors (m is up to you), each vector represented as a list, such that the solution set of

$$\begin{aligned} \boldsymbol{a}_1 \cdot \boldsymbol{x} &= 0 \\ &\vdots \\ \boldsymbol{a}_m \cdot \boldsymbol{x} &= 0 \end{aligned}$$

is Span $\{[1,1,1]\}$.

Problem 3: Consider the linear system over $\mathbb{R}$:

$$\begin{aligned} [1,1,1] \cdot \boldsymbol{x} &= 1 \\ [1,1,-1] \cdot \boldsymbol{x} &= 0 \end{aligned}$$

Find a vector $\boldsymbol{u}$ and a set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ of vectors (n is up to you), each vector represented as a list, such that the solution set of the linear system is $\boldsymbol{u}$ + Span $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$.

Problem 4: Give a list $[\beta_1, \ldots, \beta_m]$ of real numbers (m is up to you) and a list $[\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m]$ of vectors

(represented as lists) over $\mathbb{R}$ such that the solution set of the linear system

$$\begin{aligned} \boldsymbol{a}_1 \cdot \boldsymbol{x} &= \beta_1 \\ &\vdots \\ \boldsymbol{a}_m \cdot \boldsymbol{x} &= \beta_m \end{aligned}$$

is

$$[1, 2, 3] + \text{Span } \{[1, 1, 1]\}$$

Problem 5: Find a set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ of vectors (n is up to you), each represented as a list, whose affine hull equals

$$[4, 5, 6] + \text{Span } \{[-3, -2, -1], [7, 8, 0]\}$$

Problem 6: Find a vector $\boldsymbol{u}$ and a set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ of vectors (n is up to you), each represented by a list, such that $\boldsymbol{u} + \text{Span } \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ equals the affine hull of

$$\{[256, 512, 1024], [3.14159, 2.718281828, 53], [1, 10, 100]\}$$