Expectation, Continued

Another way to write the expectation of a random variable $X$ is
\[ E[X] = \sum_{k \in \mathbb{R}} k \cdot p(X = k). \]
(Proven.)

Variance

Definition: The variance of a random variable $X$, denoted $V[X]$, is the expected value of the square difference of $X$ and the expectation of $X$, i.e.
\[ V[X] = E[(X - E[X])^2] = \sum_{\omega \in S} (X(\omega) - E[X])^2 p(\omega). \]

Definition: The standard deviation of a random variable $X$ is the square root of the variance of $X$.

Proposition: The expected value of a constant $c$ is
\[ E[c] = c. \]
(Proven.)

Proposition: For a random variable $X$, we have
\[ V[X] = E[X^2] - E[X]^2. \]
(Proven.)

Definition: Two random variable $X$ and $Y$ are independent if
\[ p(X = i, Y = j) = p(X = i)p(Y = j) \]
for all $i, j \in \mathbb{R}$. (Not covered in class.)

Proposition: Suppose we have independent random variables $X$ and $Y$. Then
\[ E[XY] = E[X]E[Y]. \]
(Not covered in class.)

Proposition: Suppose we have $n$ independent random variables $X_1, \ldots, X_n$, then
\[ V[X_1 + X_2 + \ldots + X_n] = V[X_1] + V[X_2] + \ldots + V[X_n]. \]
(Not proven.)