Note: For the following definitions, we assume some probability space \((S, p)\).

**Definition:** Given two events \(A, B \subseteq S\) where \(p(B) > 0\), we define the *conditional probability of \(A\) given \(B\)* as
\[
p(A \mid B) = \frac{p(A \cap B)}{p(B)}.
\]

**Definition:** Two events \(A, B \subseteq S\) are *independent* if
\[
p(A \cap B) = p(A)p(B).
\]
This is equivalent to the condition
\[
p(A \mid B) = p(A).
\]

**Definition:** Events \(A_1, \ldots, A_n \subseteq S\) are *independent* if for all subsets \(T \subseteq \{1, \ldots, n\}\), we have
\[
p\left(\bigcap_{j \in T} A_j\right) = \prod_{j \in T} p(A_j).
\]

**Bayes Rule:**
\[
p(A \mid B) = \frac{p(B \mid A)p(A)}{p(B)}.
\]