Binomial Theorem: The coefficients of the terms in the polynomial \((x + y)^n\) are binomial coefficients, i.e.

\[
(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.
\]

Properties of the Binomial Coefficient:

- \(\binom{n}{k} = \binom{n}{n-k}\)
- \(\sum_{k=0}^{n} \binom{n}{k} = 2^n\)
- \(\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}\)

Proposition: The number of ways to write a positive integer \(m\) as a sum \(m_1 + m_2 + \ldots + m_r\) of \(r\) non-negative integers (where the order of the \(m_i\) matters) is

\[
\binom{m + r - 1}{r - 1}.
\]