Definition: A relation $R$ on the sets $A$ and $B$ is a subset of the Cartesian product $A \times B$. A relation $R$ on the set $A$ is a subset of the Cartesian product $A \times A$. Notationally, if an ordered pair $(a, b)$ is in the relation $R$, we can write $(a, b) \in R$ or $aRb$.

Definition: A relation $R$ on $A$ is reflexive if $\forall a \in A, (a, a) \in R$.

Definition: A relation $R$ on $A$ is symmetric if $\forall a, b \in A$, we have that $(a, b) \in R \Rightarrow (b, a) \in R$.

Definition: A relation $R$ on $A$ is transitive if $\forall a, b, c \in A$, we have that $((a, b) \in R$ AND $(b, c) \in R) \Rightarrow (a, c) \in R$.

Definition: An equivalence relation is a relation that is reflexive, symmetric, and transitive.

Definition: A partition of a set $A$ is a collection of subsets $B_1, \ldots, B_k$ of $A$ s.t. every element of $A$ is in some subset $B_i$, and $B_i \cap B_j = \emptyset \ \forall i, j$. We say that such a collection of blocks $B_1, \ldots, B_k$ partitions $A$.

Definition: Let $R$ be an equivalence relation on $A$. Then the equivalence class of $a \in A$, denoted $[a]_R$, is $\{x \mid x \in A, (x, a) \in R\}$.

Proposition: Given a partition $B_1, \ldots, B_k$ on a set $A$, the blocks $B_i$ are the equivalence classes of some equivalence relation on $A$.

Proposition: The equivalence classes of a relation $R$ on $A$ form a partition of $A$. 