Midterm 1 Review

Due: February 24/25, 2016

All homeworks are due at 12:55 PM in the CS22 bin on the CIT second floor, next to the Fishbowl.

Include our cover sheet or equivalent, write your Banner ID (but not your name or your CS login) on each page of your homework, label all work with the problem number, and staple the entire handin before submitting.

Be sure to fully explain your reasoning and show all work for full credit. Consult the style guide for more information.

Problem 1

Prove the following equality using the element method: \( X \cap Y = (X \cup Y) \setminus (X^c \cup Y^c) \)

Problem 2

Consider two equivalence relations, \( P \) and \( Q \), on a set \( S \).

a. Is \( R_1 = P \cap Q \) an equivalence relation on \( S \)? Prove your answer. If \( R_1 \) is an equivalence relation on \( S \), what can we say about the number of equivalence classes that \( R_1 \) has, relative to \( P \) and \( Q \)?

b. Is \( R_2 = P \cup Q \) an equivalence relation on \( S \)? Prove your answer. If \( R_2 \) is an equivalence relation on \( S \), what can we say about the number of equivalence classes that \( R_2 \) has, relative to \( P \) and \( Q \)?

Problem 3

Suppose \( f \) is a bijection between two sets \( A \) and \( B \). Then \( x, y \in A \) gives us \( f(x), f(y) \in B \). Prove that bijections preserve equivalence relations. That is, if \( R \) is an equivalence relation on \( A \), then \( R' \) is an equivalence relation on \( B \), where \( R' \) is defined as the relation such that \( f(x)R'f(y) \) if and only if \( xRy \), where \( x, y \in A \), (therefore \( f(x), f(y) \in B \)).
Problem 4

Fry is building a new hallway in the Planet Express, and he wants to know how to tile it. He knows that the hallway will be two units wide, but doesn’t know how long it will be. Your task is to find out how many ways to tile the hallway, using only $2 \times 2$, $2 \times 1$, and $1 \times 2$ tiles, pictured below.

Prove by induction that the number of ways to tile a $2 \times n$ hallways is $\frac{2^{n+1} + (-1)^n}{3}$.

Problem 5

Michael is counting up the total cent value of the Employee of the Month award and realizes that it is possible to make any positive multiple of 5 cents except for 5 and 15 using only quarters and dimes. Prove that he is correct (for once) using strong induction.