Problem Session 7
Due: Wed, March 16, 2016, Sun, March 20, 2016

All homeworks are due at 12:55 PM in the CS22 bin on the CIT second floor, next
to the Fishbowl.

Include our cover sheet or equivalent, write your Banner ID (but not your name or
your CS login) on each page of your homework, label all work with the problem
number, and staple the entire handin before submitting.

Be sure to fully explain your reasoning and show all work for full credit. Consult the
style guide for more information.

Problem 1

Create Truth Tables for each of the following

a. \((p \land \lnot q) \lor \lnot p\)
b. \((p \lor \lnot q) \land (\lnot p \lor q)\)
c. \((p \land (q \lor r))\)
d. \((p \rightarrow q) \rightarrow r\)

Solution

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>((p \land \lnot q) \lor \lnot p)</th>
<th>((p \lor \lnot q) \land (\lnot p \lor q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

1
### Problem 2

Let $S = \{p_1, p_2, \ldots, p_n\}$ be a set of propositions.

Consider propositions formed from the elements of $S$ in the following way:

a. You can use any $p_i$ as well as the negation of any $p_i$.

b. You can Or together any combination of $p_i$ and $\neg p_i$. For example, the proposition $(p_1 \lor p_3 \lor \neg p_7)$ can be formed this way.

c. You can And together terms of the form from (2). For example,

$$\neg(p_1 \lor p_2 \lor p_7) \land (\neg p_2 \lor p_1) \land (p_6 \lor p_1) \land (\neg p_3 \lor p_2)$$

can be formed this way.

The following is a short list of things that are not in the correct form:

- $p_1 \Rightarrow p_2$
- $(p_1 \land p_3) \lor (p_1 \land \neg p_4)$
- $\neg(p_1 \lor p_2)$
Express the following propositions over the set $S = \{p, q, r, s\}$ in the given form, showing all steps.

d. $(p \Rightarrow q) \oplus r$

e. $p \Rightarrow (q \land (r \Rightarrow \neg s))$

Solution

a.  

$(\neg p \lor q) \oplus r$

$(\neg p \lor q) \lor r) \land \neg((\neg p \lor q) \land r)$

$(\neg p \lor q) \lor r) \land \neg(\neg(\neg p \lor q) \lor \neg r)$

$(\neg p \lor q) \lor r) \land ((p \land \neg q) \lor \neg r)$

$(\neg p \lor q) \lor r) \land (p \lor \neg r) \land (\neg q \lor \neg r)$

b.  

$p \Rightarrow (q \land (r \Rightarrow \neg s))$

$(q \land (r \Rightarrow \neg s)) \lor \neg p$

$(\neg p \lor q) \land (\neg p \lor (r \Rightarrow \neg s))$

$(\neg p \lor q) \land (\neg p \lor \neg r \lor \neg s)$

Problem 3

State the converse, inverse, negation, and contrapositive of each of the following statements:

a. If Jim pranks Dwight, Pam laughs.

b. The Dunder Mifflin Paper Company is never a dull place to work.

c. You are enjoying yourself if you are watching The Office.

Solution

a. If Jim pranks Dwight, Pam laughs.

Converse: If Pam laughs, Jim pranks Dwight.
Inverse: If Jim does not prank Dwight, Pam does not laugh.
Negation: Jim pranks Dwight and Pam does not laugh.
Contrapositive: If Pam does not laugh, Jim does not prank Dwight.

b. The Dunder Mifflin Paper Company is never a dull place to work.
Converse: If a workplace is never dull, it is Dunder Mifflin Paper Company.
Inverse: If a workplace is not the Dunder Mifflin Paper Company, it is sometimes a dull place to work.
Negation: The Dunder Mifflin Paper Company is sometimes a dull place to work.
Contrapositive: If a workplace is sometimes dull, it is not the Dunder Mifflin Paper Company.

c. You are enjoying yourself if you are watching The Office.
Converse: If you are enjoying yourself, you are watching the office.
Inverse: If are not enjoying yourself, you are not watching the Office.
Negation: You are watching the office and not enjoying your self.
Contrapositive: If you are not enjoying yourself, you are not watching the office.

Problem 4

It’s promotion time at the Office, and Jim isn’t prepared. Michael has decided to ignore emotion so all promotions are on the basis of ability to solve a single logic problem. Help Jim solve it, and explain your reasoning.

Which of the following is correct? There is exactly one answer.

a) none of the below  
b) none of the below  
c) one of the below  
d) all of the below  
e) none of the above  
f) all of the above
Solution

(b) is the correct answer. If (a) is the correct answer, none of the letters below it is correct. However, since (b) is below (a), if none of the below a are correct, none below (b) are correct. This means that b is correct. We have therefore reached a contradiction since b being correct invalidates the correctness of (a). If (b) is correct, c-f are incorrect. (c) cannot be correct because, if it were, then it and one of those below it would be correct. Since there can only be one solution, this is not right, so (c) is incorrect. (d) cannot be correct because, again, if it were correct, more than one answer would be correct. (e) cannot be correct because if it were, then c would be correct because one of those below (c), (e), would be correct. This would result in a contradiction since (e) says none of the above are correct. (f) cannot be correct for the same reasoning as (c) and (d). As shown above, all c-f are incorrect. Therefore (b) is correct.

Problem 5

Draw a circuit for the following logical expressions. Simplify the expressions first if possible.

a. \( \neg(\neg x_1 \land x_2) \land (x_1 \lor \neg x_3) \land (x_2 \lor x_4) \)

b. \( (x_1 \land (x_2 \lor \neg x_2) \land (x_3 \lor \neg x_1)) \lor ((x_1 \lor x_2) \land (\neg x_2 \land x_4)) \)
Solution

Part a.

Part b.