Homework 1
Due: Wednesday, February 10, 2016

All homeworks are due at 12:55 PM in the CS22 bin on the CIT second floor, next to the Fishbowl.

Include our cover sheet or equivalent, write your Banner ID (but not your name or your CS login) on each page of your homework, label all work with the problem number, and staple the entire handin before submitting.

Be sure to fully explain your reasoning and show all work for full credit. Consult the style guide for more information.

Problem 1

Prove that the sum of any four consecutive integers is even.

Problem 2

For this problem, let

\[ A = \{ \emptyset, 0, \{ \emptyset \}, \{0, \emptyset \} \} \]
\[ B = \{ \emptyset, \{ \emptyset \}, \{0, \emptyset, 1 \} \} \]
\[ C = \{ x \mid \exists y \in \mathbb{Z} \text{ s.t. } y^2 = x, x < 10 \} \]

Find:

- a. \( A \cup B \)
- b. \( A \cap B \)
- c. \( A \setminus B \)
- d. \( \mathcal{P}(B) \), the power set of \( B \)
- e. \( |\mathcal{P}(A \cup B)| \)
- f. \( C \setminus (A \cup B) \)
- g. \( \{ x \mid x \in B, |x| \notin C \} \)
Problem 3

Due to corporate downsizing, Michael Scott needs to choose some of the employees at Dunder Mifflin’s Scranton office to keep, and must lay off all the rest. He wants to keep an employee if (and only if): (i) either he finds them competent or they are nice to him, and (ii) they are not Toby.

Consider the following sets:

- Employees: $E = \{\text{Andy, Angela, Dwight, Jim, Kevin, Pam, Ryan, Toby}\}$
- Competent: $C = \{\text{Jan, Pam, Ryan}\}$
- Nice: $N = \{\text{Andy, Dwight, Pam, Toby}\}$
- Toby: $T = \{\text{Toby}\}$

For parts a through e, translate the phrase into proper set notation (using the above four sets) and list the elements in the set. For example, the answer for “competent employees” would be

\[ E \cap C = \{\text{Pam, Ryan}\} \]

a. Employees who are not nice.

b. Nice people who are not competent.

c. Employees who are either competent or Toby.

d. Nice people who are competent and employees.

e. The list of people that Michael wants to keep.

Problem 4

Let $A = \{\emptyset, 2\}$, $B = \{4, 5, 6\}$, $C = \{2, 4, 6\}$, and $D = \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$.

Find the cardinalities of the following sets:

a. $A$

b. $\{A, C, 3\}$

c. $B \cup C$

d. $A \cap C$

e. $C \setminus B$

f. $\mathcal{P}(B)$

g. $\mathcal{P}(\mathcal{P}(B))$
h. $B \times D$

i. $\mathcal{P}(\mathcal{P}(A) \times D)$

Note: Your answer may be given as a power of 2.

Problem 5

For each of the following, either prove the statement using the element method, or give a counterexample.

a. $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

b. $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

Problem 6

Let $x$ be a nonzero rational, let $y$ and $z$ be irrational, and prove or give a counterexample for each of the following statements. If you provide a counterexample, be sure to justify that the example makes the claim false. You may use, without proof, the fact that $\sqrt{2}$ is irrational. However, you may not assume other numbers are irrational without proof!

$\forall x \in \mathbb{Q}, y$ and $z \in (\mathbb{R} \setminus \mathbb{Q})$:

a. $xy$ is irrational.

b. $x + yz$ is irrational.

c. $x + yz$ is rational.