Sorting & Master Theorem

CS16: Introduction to Data Structures & Algorithms
Seny Kamara - Spring 2017
Outline

- Motivation
- Quadratic Sorting
  - Selection sort
  - Insertion sort
- Linearithmic Sorting
  - Merge Sort
  - Master Theorem
  - Quick Sort
- Comparative sorting lower bound
- Linear Sorting
  - Radix Sort
The Problem

- Turn this

\[
\begin{array}{cccccccccccc}
10 & 19 & 7 & 4 & 3 & 21 & 10 & 23 & 24 & 18 & 1 & 8 & 23 & 1 & 12 \\
\end{array}
\]

- Into this

\[
\begin{array}{cccccccccccc}
1 & 1 & 3 & 4 & 7 & 8 & 10 & 10 & 12 & 18 & 19 & 21 & 23 & 23 & 24 \\
\end{array}
\]

- as efficiently as possible
Sorting Algorithms

- There are many ways to sort arrays
  - Iterative vs. recursive
  - in-place vs. not-in-place
  - comparison-based vs. non-comparative

- In-place algorithms
  - transform data structure w/ small (i.e., $O(1)$) extra storage
  - For sorting: array is overwritten by output instead of creating new array

- Most sorting algorithms in 16 are comparison-based
  - main operation is comparison
  - but not all (e.g., Radix sort)
“In-Placeness”

- Reversing an array

```javascript
function reverse(A):
    n = A.length
    B = array of length n
    for i = 0 to n - 1:
        B[n-1-i] = A[i]
    return B
```

Not in-place!

```javascript
function reverse(A):
    n = A.length
    for i = 0 to n/2:
        temp = A[i]
        A[n-1-i] = temp
    return statement not needed
```
in-place
Properties of In-Place Solutions

- Harder to write :-(
- Use less memory :-)
- Even harder to write for recursive algorithms :-(
- Tradeoff between simplicity and efficiency
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Selection Sort

- Usually iterative and in-place
- Divides input array into two logical parts
  - elements already sorted
  - elements that still need to be sorted
- Selects smallest element & places it at index 0
  - then selects second smallest & places it in index 1
    - then the third smallest at index 2, etc..
Selection Sort

- **Advantages**
  - Very simple
  - Memory efficient if in-place (swaps elements in array)

- **Disadvantages**
  - Slow: $O(n^2)$
Selection Sort

- Iterate through positions
- At each position
  - store smallest element from remaining set
Selection Sort

\[
\text{function } \text{selection\_sort}(A): \\
\text{n} = A.\text{length} \\
\text{for } i = 0 \text{ to } n-2: \\
\quad \text{min} = \text{argmin}(A[i:n-1]) \\
\quad \text{swap } A[i] \text{ with } A[\text{min}] 
\]
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Insertion Sort

- Usually iterative and in-place
- Compares each item w/ all items before it...
  - ...and inserts it into correct position

Advantages

- Works really well if items partially sorted
- Memory efficient if in-place (swaps elements in array)

Disadvantages

- Slow: $O(n^2)$
Insertion Sort

- Compares each item with all items before it...
- ...and inserts it into correct position

Note: 23 > 22 so don’t need to keep checking since rest is already sorted
function `insertion_sort(A)`:

```python
n = A.length
for i = 1 to n-1:
    for j = i down to 1:
        if a[j] < a[j-1]:
            swap a[j] and a[j-1]
        else:
            break  # out of the inner for loop
            # this prevents double checking the
            # already sorted portion
```
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  - **Merge Sort**
    - Master Theorem
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  - Radix Sort
Divide & Conquer

- Algorithmic design paradigm
  - divide: divide input $S$ into disjoint subsets $S_1, \ldots, S_k$
  - recur: solve sub-problems on $S_1, \ldots, S_k$
  - conquer: combine solutions for $S_1, \ldots, S_k$ into solution for $S$
  - Base case is usually sub-problem of size 1 or 0
Merge Sort

- Sorting algorithm based on divide & conquer
- Like quadratic sorts
  - comparative
- Unlike quadratic sorts
  - recursive
  - linearithmic $O(n \log n)$
Merge Sort

- Merge sort on n-element sequence $S$
  - divide: divide $S$ into disjoint subsets $S_1$ and $S_2$
  - recur: recursively merge sort $S_1$ and $S_2$
  - conquer: merge $S_1$ and $S_2$ into sorted sequence
- Suppose we want to sort
  - 7, 2, 9, 4, 3, 8, 6, 1
Merge Sort Recursion Tree

7 2 9 4 | 3 8 6 1

20
Merge Sort Recursion Tree
Merge Sort Recursion Tree
Merge Sort Recursion Tree
Merge Sort Recursion Tree
Merge Sort Recursion Tree

- 7 2 9 4 | 3 8 6 1
- 7 2 | 9 4
- 7 | 2 → 2 7
- 7 → 7, 2 → 2
- 9 → 9, 4 → 4
- 3 → 3, 8 → 8, 6 → 6, 1 → 1
Merge Sort Recursion Tree

7 2 9 4 | 3 8 6 1

7 2 | 9 4

7 → 7 2 → 2

9 → 9 4 → 4

3 → 3 8 → 8 6 → 6 1 → 1

7 → 7 2 → 2 9 → 9 4 → 4
Merge Sort Recursion Tree
Merge Sort Recursion Tree
Merge Sort Recursion Tree
Merge Sort Pseudo-Code

function mergeSort(A):
    n = A.length
    if n <= 1:
        return A

    mid = n/2
    left = mergeSort(A[0...mid-1])
    right = mergeSort(A[mid...n-1])
    return merge(left, right)
function merge(A, B):
    result = []
    aIndex = 0
    bIndex = 0
    while aIndex < A.length and bIndex < B.length:
        if A[aIndex] <= B[bIndex]:
            result.append(A[aIndex])
            aIndex++
        else:
            result.append(B[bIndex])
            bIndex++
    if aIndex < A.length:
        result = result + A[aIndex:end]
    if bIndex < B.length:
        result = result + B[bIndex:end]
    return result
Merge Sort

Activity #1

2 min
Merge Sort

Activity #1

2 min
Merge Sort

Activity #1

1 min
Merge Sort

Activity #1
Merge Sort Recurrence Relation

- Merge sort steps
  - Recursively merge sort left half
  - Recursively merge sort right half
  - Merge both halves

- $T(n)$: time to merge sort input of size $n$
  - $T(n) = \text{step 1} + \text{step 2} + \text{step 3}$
  - Steps 1 & 2 are merge sort on half input so $T(n/2)$
  - Step 3 is $O(n)$
Merge Sort Recurrence Relation

- General case
  \[ T(n) = T \left( \frac{n}{2} \right) + T \left( \frac{n}{2} \right) + O(n) = 2 \cdot T \left( \frac{n}{2} \right) + O(n) \]

- Base case
  \[ T(1) = c \]
Merge Sort Recurrence Relation

- Plug & chug
  
  \[ T(1) = c_1 \]
  
  \[ T(2) = 2 \cdot T(1) + 2 = 2c_1 + 2 \]
  
  \[ T(4) = 2 \cdot T(2) + 4 = 2(2c_1 + 2)4 = 4c_1 + 8 \]
  
  \[ T(8) = 2 \cdot T(4) + 8 = 2(4c_1 + 8) + 8 = 8c_1 + 24 \]
  
  \[ T(16) = 2 \cdot T(8) + 16 = 2(8c_1 + 24) + 16 = 16c_1 + 64 \]

- Solution
  
  \[ T(n) = nc_1 + n \log n = O(n \log n) \]
Analysis of Merge Sort

- Merge sort recursive tree is perfect binary tree so has height $O(\log n)$
- At each depth $k$: need to split and merge $2^k$ sequences of size $n/2^k$
  - work at each depth is $O(n)$

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<tr>
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<td>4</td>
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<tr>
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<td>$\vdots$</td>
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<tr>
<td>$k$</td>
<td>$2^k$</td>
<td>$n/2^k$</td>
</tr>
</tbody>
</table>
Analysis of Merge Sort

- To determine that Merge sort was $O(n \log n)$
  - Used plug and chug to guess a solution
  - Prove that $O(n \log n)$ is correct (e.g., using induction)
- Can be a lot of work
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The Master Theorem

- Solves large class of recurrence relations
  - we will learn how to use it but not its proof
  - See Dasgupta et al. p. 58-60 for proof
- Let $T(n)$ be a monotonically-increasing function of form
  \[
  T(n) = a \cdot T\left(\frac{n}{b}\right) + \Theta(n^d)
  \]
  - $a$: number of sub-problems
  - $n/b$: size of each sub-problem
  - $n^d$: work to prepare sub-problems & combine their solutions
The Master Theorem

- If $a \geq 1$, $b > 1$, $d \geq 0$, then
  - $T(n) = \Theta(n^d)$ if $a < b^d$
  - $T(n) = \Theta(n^d \log n)$ if $a = b^d$
  - $T(n) = \Theta(n^{\log_b a})$ if $a > b^d$

- Applying Master Theorem to merge sort
  - Rec. relation: $T(n) = 2T(n/2) + O(n^1)$
  - so $a = 2$, $b = 2$ and $d = 1$ and $a = b^d$
  - and $T(n) = \Theta(n^d \log n) = \Theta(n^1 \log n) = \Theta(n \log n)$
Master Theorem

\[ T(n) = a \cdot T \left( \frac{n}{b} \right) + \Theta(n^d) \]

- \( T(n) = \Theta(n^d) \) if \( a < b^d \)
- \( T(n) = \Theta(n^d \log n) \) if \( a = b^d \)
- \( T(n) = \Theta(n^{\log_b a}) \) if \( a > b^d \)

Activity #2+3
Master Theorem

\[ T(n) = a \cdot T \left( \frac{n}{b} \right) + \Theta(n^d) \]

- \( T(n) = \Theta(n^d) \) if \( a < b^d \)
- \( T(n) = \Theta(n^d \log n) \) if \( a = b^d \)
- \( T(n) = \Theta(n^{\log_b a}) \) if \( a > b^d \)

Activity #2+3
Master Theorem

\[ T(n) = a \cdot T \left( \frac{n}{b} \right) + \Theta(n^d) \]

- \[ T(n) = \Theta(n^d) \text{ if } a < b^d \]
- \[ T(n) = \Theta(n^d \log n) \text{ if } a = b^d \]
- \[ T(n) = \Theta(n^{\log_b^a}) \text{ if } a > b^d \]

Activity #2+3

1 min
Master Theorem

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + \Theta(n^d)$$

- $T(n) = \Theta(n^d)$ if $a < b^d$
- $T(n) = \Theta(n^d \log n)$ if $a = b^d$
- $T(n) = \Theta(n^{\log_b a})$ if $a > b^d$

Activity #2+3
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Quicksort

- Randomized sorting algorithm
- Based on divide-and-conquer
  - divide: pick random element (called pivot) and partition set into
    - \( L \): elements less than \( x \)
    - \( E \): elements equal to \( x \)
    - \( G \): elements larger than \( x \)
  - recur: quicksort \( L \) and \( G \)
  - conquer: join \( L \), \( E \) and \( G \)
Quicksort

Activity #4

2 min
Quicksort

2 min

Activity #4
Quicksort

Activity #4

1 min
Quicksort

Activity #4
Quicksort Example

random pivot

7 2 9 4 3 7 6 1
Quicksort Example
Quicksort Example
Quicksort Example
Quicksort Example

7 2 9 4 3 7 6 1

2 4 3 1 → 1 2 3 4

1 → 1

4 3 → 3 4

4 → 4

7 9 7
Quicksort Example
Quicksort Example
function **quick_sort** (A):
    if A.length ≤ 1
        return A

    pivot = random element from A
    L = [], E = [], G = []

    for each x in A:
        if x < pivot:
            L.append(x)
        else if x > pivot:
            G.append(x)
        else E.append(x)

    return quick_sort(L) + E + quick_sort(G)
Worst-Case Running Time

- Worst-case for Quicksort
  - when pivot is (unique) min or max element
  - Either $L$ or $G$ has size $n-1$ and other has size 0
  - Runtime is proportional to
    - $n + (n-1) + (n-2) + ... + 2 + 1$
  - Which is $O(n^2)$

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<tr>
<td>2</td>
<td>$n-2$</td>
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<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$n-1$</td>
<td>1</td>
</tr>
</tbody>
</table>
Expected Runtime of Quicksort

- Assume there are no duplicates
  - if there are then we have even less recursive calls
- At each level of recursion, Quicksort can make \( n \) different & unique recursive calls depending on the chosen split/pivot
  - \(|L| = 0\) and \(|G| = n-1\)
  - \(|L| = 1\) and \(|G| = n-2\)
  - ...
  - \(|L| = n\) and \(|G| = 0\)
- Since there are \( n \) possible splits...
- ...and since the split is chosen uniformly at random...
- ...each split is chosen with probability \( 1/n \)
Expected Runtime of Quicksort

- Each split is chosen with probability $1/n$
- So expected running time is

$$T(n) = n + \frac{1}{n} \cdot \left( T(0) + T(n-1) \right) + \cdots + \frac{1}{n} \cdot \left( T(n-1) + T(n-1 - (n-1)) \right)$$

$$= n + \frac{1}{n} \cdot \sum_{i=0}^{n-1} \left( T(i) + T(n - 1 - i) \right)$$

- Solution is $T(n) = 2n \ln n = 1.39 \cdot n \log_2 n = O(n \log n)$
function `quick_sort`(A):
    if A.length ≤ 1
        return A
    pivot = random element from A
    L = [], E = [], G = []
    for each x in A:
        if x < pivot:
            L.append(x)
        else if x > pivot:
            G.append(x)
        else E.append(x)
    return quick_sort(L) + E + quick_sort(G)
In-Place Quicksort

function partition(A, low, high):
    pivotIndex = random index between low and high
    pivotValue = A[pivotIndex]
    swap A[pivotIndex] and A[high]  # move pivot to end
    currIndex = low
    for i from low to high − 1:
        if A[i] <= pivotValue :
            swap A[i] and A[currIndex]
            currIndex++
    swap A[currIndex] and A[high]   # move the pivot back
    return currIndex
In-Place Quicksort

```python
function quick_sort(A, low, high):
    if low < high:
        pivotIndex = partition(A, low, high)
        quicksort(A, low, pivotIndex - 1)
        quicksort(A, pivotIndex + 1, high)
```
Merge Sort vs. Quicksort

- Merge sort is worst-case $O(n \log n)$
- Quicksort is expected $O(n \log n)$
- Which is better?
- In practice quicksort is faster!
  - it also uses less space
  - constants are better
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How Fast Can We Sort?

- Merge sort and Quicksort are $O(n \log n)$
- Can we do better?
  - No!
  - Well kind of...

Any comparison-based sorting algorithm has to make at least $\Omega(n \log n)$ comparisons in the worst-case to sort $n$ keys.
Lower Bound on Comparative Sorting

- Viewed abstractly, a sorting algorithm
  - takes a sequence of keys $k_1, \ldots, k_n$
  - outputs a permutation of the keys that has them in order
- We can view the optimal (i.e., best possible) algorithm as a perfect binary **decision** tree
  - internal nodes do comparisons of keys
  - leaves are the correct permutation
- To sort a sequence, we traverse tree
- Worst-case number of comparisons is height of tree
Lower Bound on Comparative Sorting

- Suppose our input is X, Y, Z...
- ...and the proper order is Z, X, Y

Equal terms will be considered $\leq$ but $\geq$ would also work
Lower Bound on Comparative Sorting

- How many leaves does tree have?
  - \( n! \) because there are \( n! \) permutations of a sequence of \( n \) elements
  - A perfect binary tree with \( L \) leaves has height \( \log L \)
  - So tree with \( n! \) leaves has height \( \log(n!) \)
  - Stirling’s formula
    \[ n! = n^n e^{-n} \]
    \[ \log(n!) \geq \log(n^n e^{-n}) \]
    \[ \log(n!) \geq n \log n - n \log e \]
  - So height of tree (and # of comparisons) is \( \Omega(n \log n) \)
Non-Comparative Sorting

- Sorting functions are used on different types of inputs
  - Integers, floats, strings, arrays, other objects…
  - As long as we can compare the inputs we can use comparative sorting algorithms
- But for certain kinds of inputs, we can sometimes do better
  - example: for positive integers we can use Radix sort
Radix Sort

‣ How would you sort 258391 and 258492?
  ‣ digit by digit
  ‣ the 3 high order digits are same...
  ‣ ...so you keep going until you see 3<4 so 258391 must less than 258492
Radix Sort

- How would you sort an array of numbers between 0 and 9?
  - example: \([5, 1, 6, 2, 3, 1] \rightarrow [1, 1, 2, 3, 5, 6]\)
  - Create array of 10 buckets
  - for each number \(x\), add it to bucket at index \(x\)
  - Return concatenation of all buckets (in order)
    - print out \([1, 1] + [2] + [3] + [5] + [6]\)
  - Runtime is \(O(n)\)
Radix Sort

- Radix sort combines both approaches
- Takes advantage of
  - the "digit-iness" of integers
  - for every digit there are $O(1)$ number of options
Radix Sort

- Sort \([273, 279, 8271, 7891, 8736, 8735]\]
- Start with lowest-order digit (the 1's place)
  - add number to bucket corresponding to that digit

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- Concatenate all buckets
  - \([8271, 7891, 273, 8735, 8736, 279]\]
- Now sorted by lowest-order digit
Radix Sort

- Sort \([8271, 7891, 273, 8735, 8736, 279]\)
- Start with second lowest-order digit (the 10’s place)

  - add number to bucket corresponding to that digit

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</table>

- Concatenate all buckets

  - \([8735, 8736, 8271, 273, 279, 7891]\)
- Now sorted by second and lowest-order digit
Radix Sort

- Sort $[8735, 8736, 8271, 273, 279, 7891]$
- Start with third lowest-order digit (the 100’s place)
  - add number to bucket corresponding to that digit

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- Concatenate all buckets
  - $[8271, 273, 279, 8735, 8736, 7891]$
- Now sorted by third, second and lowest-order digit
Radix Sort

- Sort $[8271, 273, 279, 8735, 8736, 7891]$
- Start with third lowest-order digit (the 1000’s place)
  - add number to bucket corresponding to that digit

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<td></td>
<td>7891</td>
<td>8271</td>
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<td>279</td>
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<td></td>
<td></td>
<td>8735</td>
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<td></td>
<td></td>
<td>8736</td>
<td></td>
</tr>
</tbody>
</table>

- Concatenate all buckets
  - $[273, 279, 7891, 8271, 8735, 8736]$
- Now sorted by third, second and lowest-order digit
Radix Sort

function `radix_sort(A)`:
   buckets = array of 10 lists
   for place = least to most significant
       for number in A
           d = digit in number at place
           buckets[d].append(number)
       A = concatenate all buckets in order
   empty all buckets
return A

- Very efficient!
  - O(nd)
  - d is number of digits in the largest number
More on Radix Sort

- Can be applied to
  - positive integers in base 10 (we just saw this)
  - Octals (base 8)
  - Hexadecimal (base 16)
  - Strings (one bucket for every valid character)
- Number of buckets can be different at each round
- Can represent almost anything as a bit string and radix sort with two buckets
  - number of digits will dominate runtime
  - for long sequences will be very slow
Radix Sort

Activity #5

2 min
Radix Sort

Activity #5

2 min
Radix Sort

Activity #5

1 min
Radix Sort

Activity #5
# Summary of Sorting Algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Time</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection sort</td>
<td>$O(n^2)$</td>
<td>in-place</td>
</tr>
<tr>
<td></td>
<td></td>
<td>slow (good for small inputs)</td>
</tr>
<tr>
<td>Insertion sort</td>
<td>$O(n^2)$</td>
<td>in-place</td>
</tr>
<tr>
<td></td>
<td></td>
<td>slow (good for small inputs)</td>
</tr>
<tr>
<td>Merge sort</td>
<td>$O(n \log n)$</td>
<td>fast (good for large inputs)</td>
</tr>
<tr>
<td>Quick sort</td>
<td>$O(n \log n)$</td>
<td>randomized</td>
</tr>
<tr>
<td></td>
<td>expected</td>
<td>fastest (good for large inputs)</td>
</tr>
<tr>
<td>Radix sort</td>
<td>$O(nd)$</td>
<td>$d$ is number of digits in largest number</td>
</tr>
<tr>
<td></td>
<td></td>
<td>basically linear when $d$ is small</td>
</tr>
</tbody>
</table>
Readings

- Dasgupta et al.
  - **Section 2.1**: good intro to divide & conquer
  - **Section 2.2**: review of recurrence rels. & master theorem
  - **Section 2.3**: analysis of merge sort & lower bound on comparative sorting
References

- Slide #62
  - The character depicted is Raditz (sometimes called Radix) from the Anime *Dragon Ball Z*. He is the biological brother of Goku and one of the four remaining Universe 7 Saiyans.

- Slide #64
  - The RZA is the main producer and leader of the Wu-Tang Clan. He also released albums as his alter ego Bobby Digital.