Priority Queues & Heaps

CS16: Introduction to Data Structures & Algorithms
Spring 2018
Outline

- Priority Queues
  - Motivation
  - ADT
  - Implementation

- Heaps
  - insert() and upheap()
  - removeMin() and downheap()
Motivation

- Priority queues store items with various priorities

Examples

- Plane departures: some flights have higher priority than others
- Bandwidth management: real-time traffic like Skype transmitted first
Priority Queue ADT

- Stores key/element pairs
  - key determines position in queue
- `insert(key, element)`:  
  - inserts element with key
- `removeMin()`:  
  - removes pair w/ smallest key and returns element
Priority Queue Implementations

Activity #1 2 min
Priority Queue Implementations

Activity #1

2 min
Priority Queue Implementations

Activity #1

1 min
Priority Queue Implementations

Activity #1
# Priority Queue Implementation

<table>
<thead>
<tr>
<th>Implementation</th>
<th>add</th>
<th>removeMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted Array</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Sorted Array</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Unsorted Linked List</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Sorted Linked List</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Hash Table</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Heap</td>
<td>$O(\log n)$</td>
<td>$O(\log n)$</td>
</tr>
</tbody>
</table>
What is a Heap?

- Data structure that implements priority queue
- Heaps can be implemented with
  - Tree
  - Array
- Tree-based heap
Heap Properties

- Binary tree
  - each node has at most 2 children
- Each node has a priority (key)
- Heap has an order
  - min-heap: \( n.key \geq n.parent.key \)
  - max-heap: \( n.key \leq n.parent.key \)
- Left-complete
- Height of \( O(\log n) \)
Heap Properties

- To implement priority queue
  - insert key/element pair at each node
Heap — insert( )

- Need to keep track of “insertion node”
  - leaf where we will insert new node...
  - ...so we can keep heap left-complete
Heap — insert()

- Ex: insert(1)
  - replace insertion node w/ new node

Heap order violated!
Heap — upheap( )

- Repair heap: swap new element up tree until keys are sorted
- First swap fixes everything below new location
  - since every node below 6’s old location has to be at least 6…
  - …they must be at least 1
Heap — `upheap()`

- One more swap since $1 \leq 2$
- Now left-completeness and order are satisfied
Heap insert()
Heap insert()
Heap insert()
Heap insert()
Heap — upheap() Summary

- After inserting a key $k$, order may be violated
- `upheap()` restores order by
  - swapping key upward from insertion node
  - terminates when either root is reached
  - …or some node whose parent has key at most $k$

- Heap insertion has runtime
  - $O(\log n)$, why?
  - because heap has height $O(\log n)$
Heap — removeMin()

- Remove root
  - because it is always the smallest element
- How can we remove root w/o destroying heap?

![Heap diagram](image)
Heap — removeMin( )

- Instead swap root with last element & remove it
  - removing last element is easy

Order destroyed!
Heap — removeMin()

- Now swap root down as necessary

Heap is in order!
Heap — downheap() Summary

- downheap() restores order by
  - swapping key downward from root with smaller of 2 children
  - terminates when either leaf is reached or
    - ...some node whose children has key at least k
- downheap() has runtime
  - $O(\log n)$, why?
  - because heap has height $O(\log n)$
Heap removeMin()
Heap removeMin()

Activity #1

2 min
Heap removeMin()
Heap removeMin()
Summary of Heap

- `insert(key, value)`
  - insert value at insertion node
    - insertion node must be kept track of
  - `upheap()` from insertion node as necessary

- `removeMin()`
  - swap root with last item
  - delete (swapped) last item
  - `downheap()` from root as necessary
Array-based Heap

- Heap with $n$ keys can be represented w/ array of size $n+1$
- Storing nodes in array
  - Node stored at index $i$
    - left child stored at index $2i$
    - right child stored at index $2i+1$
  - Leaves & edges not stored
- Cell 0 not used

- Operations
  - insert: store new node at index $n+1$
  - removeMin: swap w/ index $n$ and remove
Finding Insertion Node

- Can be found in $O(\log n)$
- Start at last added node
- Go up until a left child or root is reached
- If left child
  - go to sibling (corresponding right child)
  - then go down left until leaf is reached

Can be done in $O(1)$ time by using additional data structure...need this for project!
## Priority Queue Implementation

<table>
<thead>
<tr>
<th>Implementation</th>
<th>add</th>
<th>removeMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted Array</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Sorted Array</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Unsorted Linked List</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Sorted Linked List</td>
<td>$O(n)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>Hash Table</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>Heap</td>
<td>$O(\log n)$</td>
<td>$O(\log n)$</td>
</tr>
</tbody>
</table>
References

- Slide #4
  - “Queue” in French means tail
  - The picture depicts the tail of a whale

- Slide #7
  - The picture is of a Transformers character named Junkheap which transforms from a waste management garbage truck

- Slide #8
  - The picture is the symbol of the Hidden Leaf Village (where the character Naruto is from)
  - The heap priorities represent the importance of the character in the village