Priority Queues & Heaps

CS16: Introduction to Data Structures & Algorithms

Spring 2020

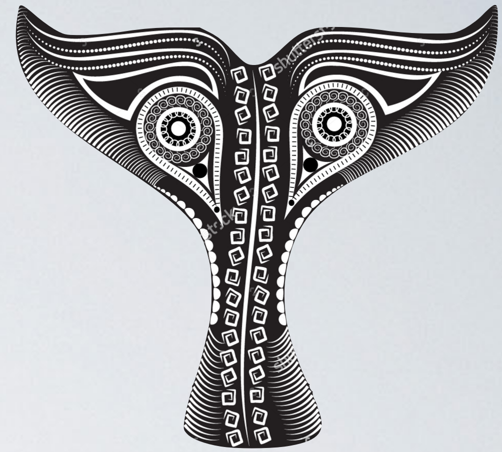
Outline

- Priority Queues
  - Motivation
  - ADT
  - Implementation
- Heaps
  - insert( ) and upheap( )
  - removeMin( ) and downheap( )

Motivation

- Priority queues store items with various priorities
- Priority queues are everywhere
  - Plane departures: some flights have higher priority than others
  - Bandwidth management: real-time traffic like Skype transmitted first
  - Student dorm room allocations
  - …

Priority Queue ADT



- Stores key/element pairs
  - key determines position in queue
- **`insert(key, element)`**:
  - inserts element with key
- **`removeMin( )`**:
  - removes pair w/ smallest key and returns element

Priority Queue Implementations

Activity #1

2 min

Priority Queue Implementations

Activity #1

2 min

Priority Queue Implementations

Activity #1

1 min

Priority Queue Implementations

Activity #1

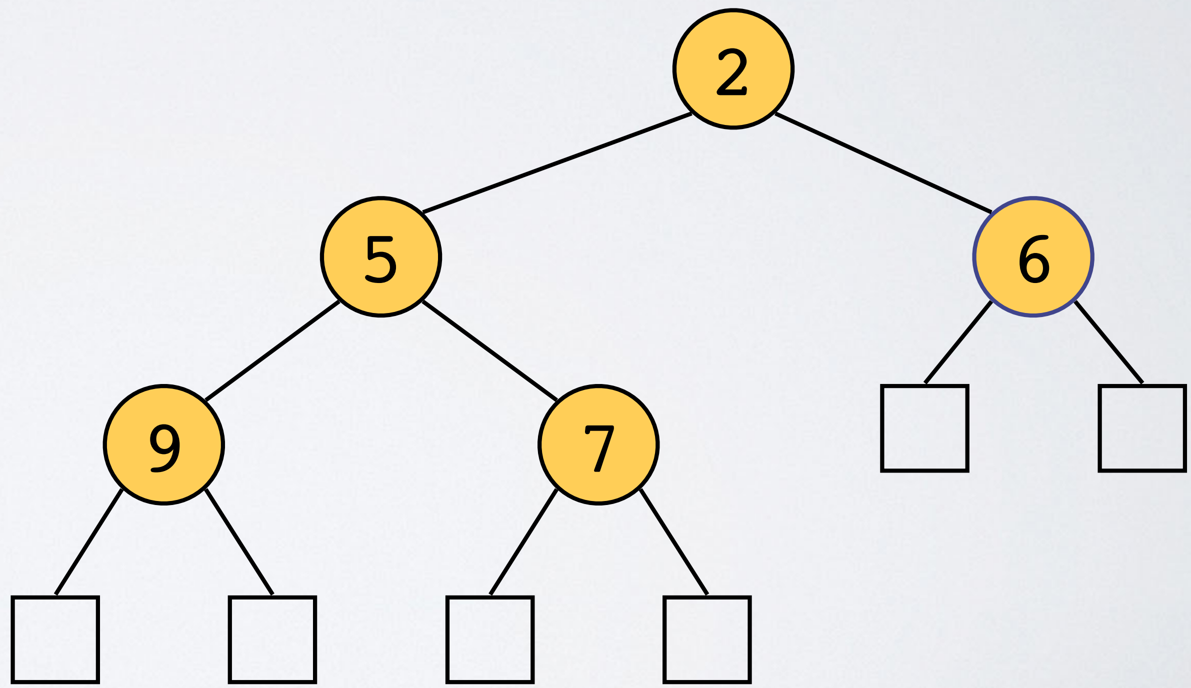
0 min

Priority Queue Implementation

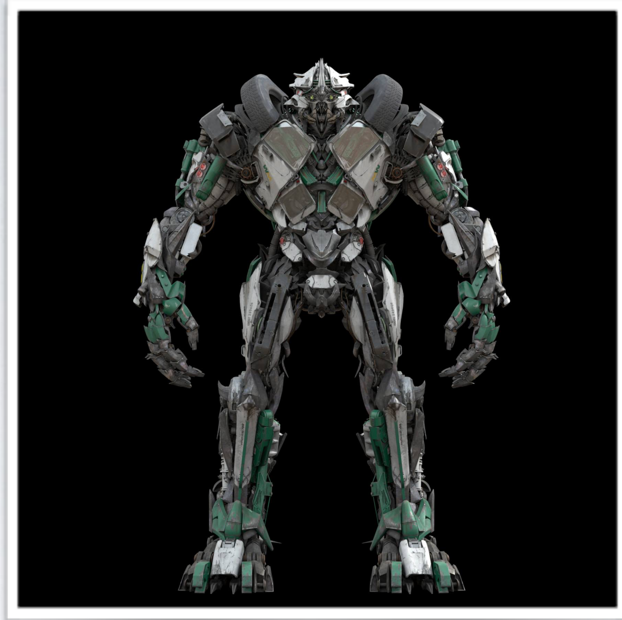
Implementation	insert	removeMin
Unsorted Array	`O(1)`	`O(n)`
Sorted Array	`O(n)`	`O(1)`
Unsorted Linked List	`O(1)`	`O(n)`
Sorted Linked List	`O(n)`	`O(1)`
Hash Table	`O(1)`	`O(n)`
Heap	`O(log n)`	`O(log n)`

What is a Heap?

- Data structure that implements priority queue
- Heaps can be implemented with
  - Tree
  - Array
- Tree-based heap



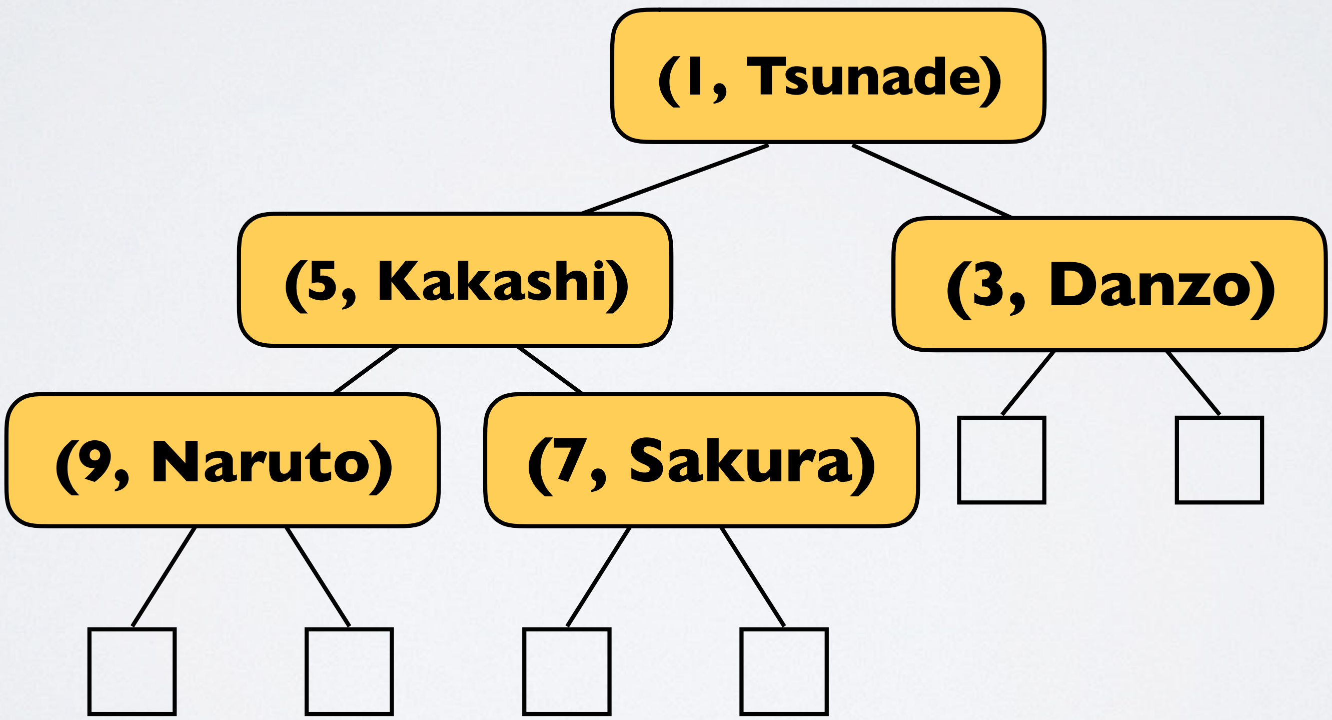
Heap Properties



- Binary tree
  - each node has at most `2` children
- Each node has a priority (key)
- Heap has an order
  - min-heap: n.parent.key ≤ n.key
  - max-heap: n.parent.key ≥ n.key
- Left-complete
- Height of `O(log n)`

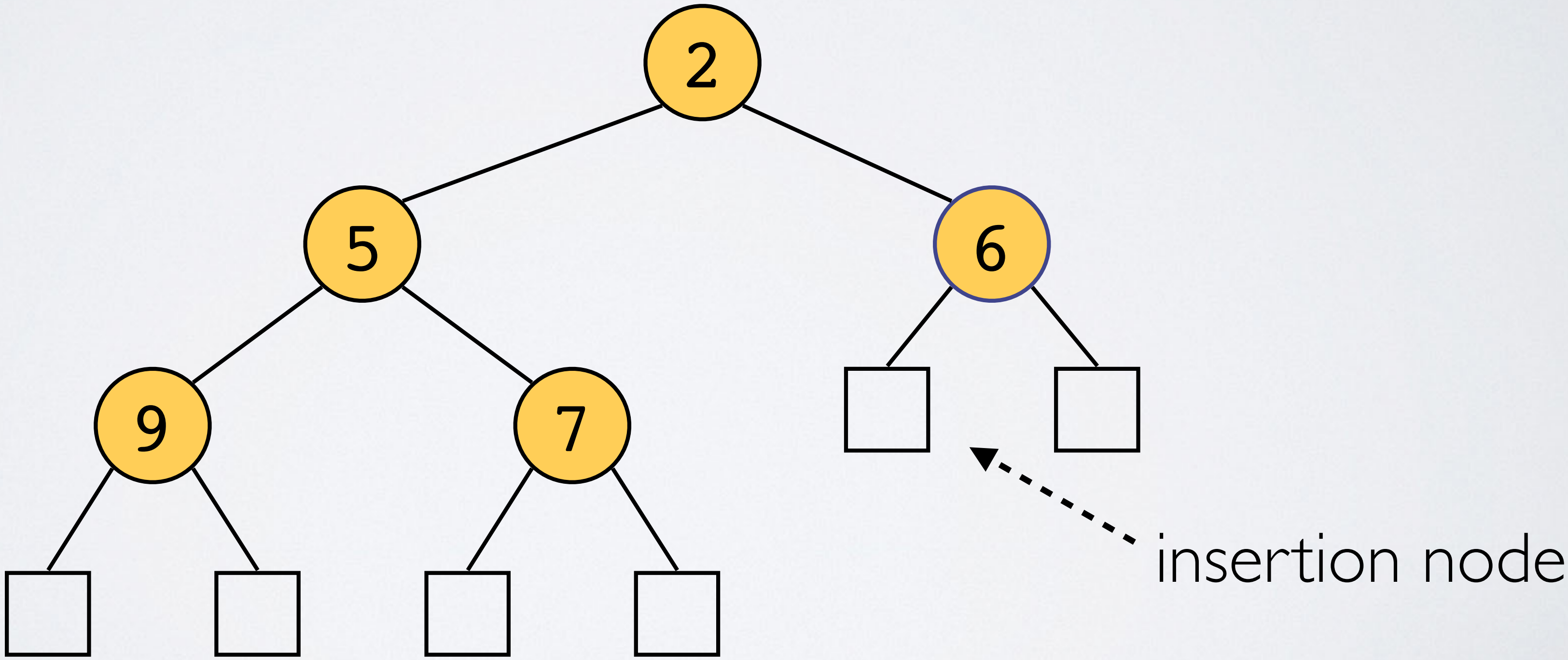
Heap Properties

- To implement priority queue
  - insert key/element pair at each node



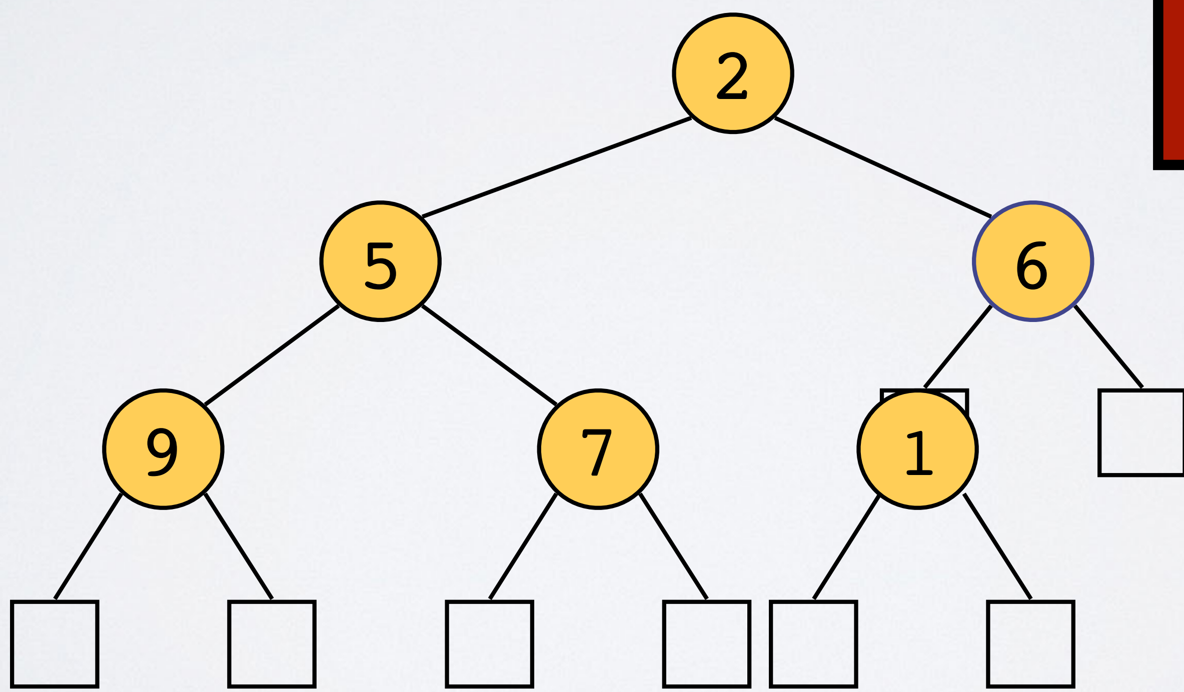
Heap: `insert`

- Need to keep track of "insertion node"
  - leaf where we will insert new node…
  - …so we can keep heap left-complete



Heap: `insert`

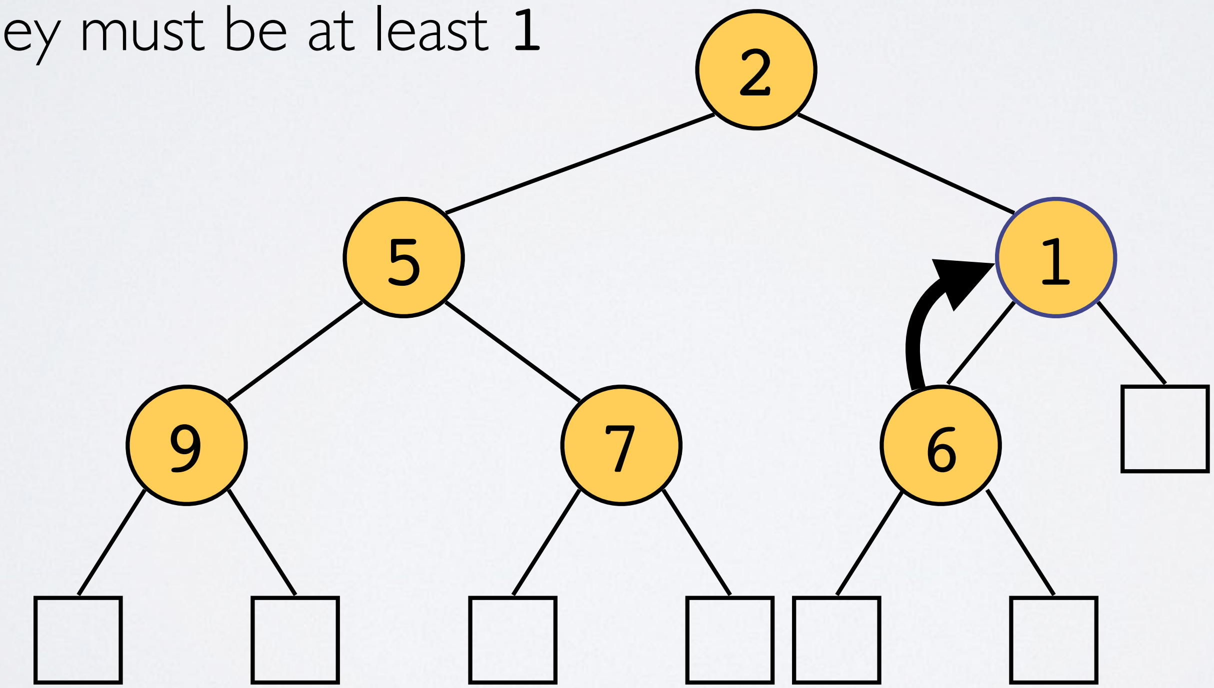
- Ex: insert(`1`)
  - replace insertion node w/ new node



Heap order violated!

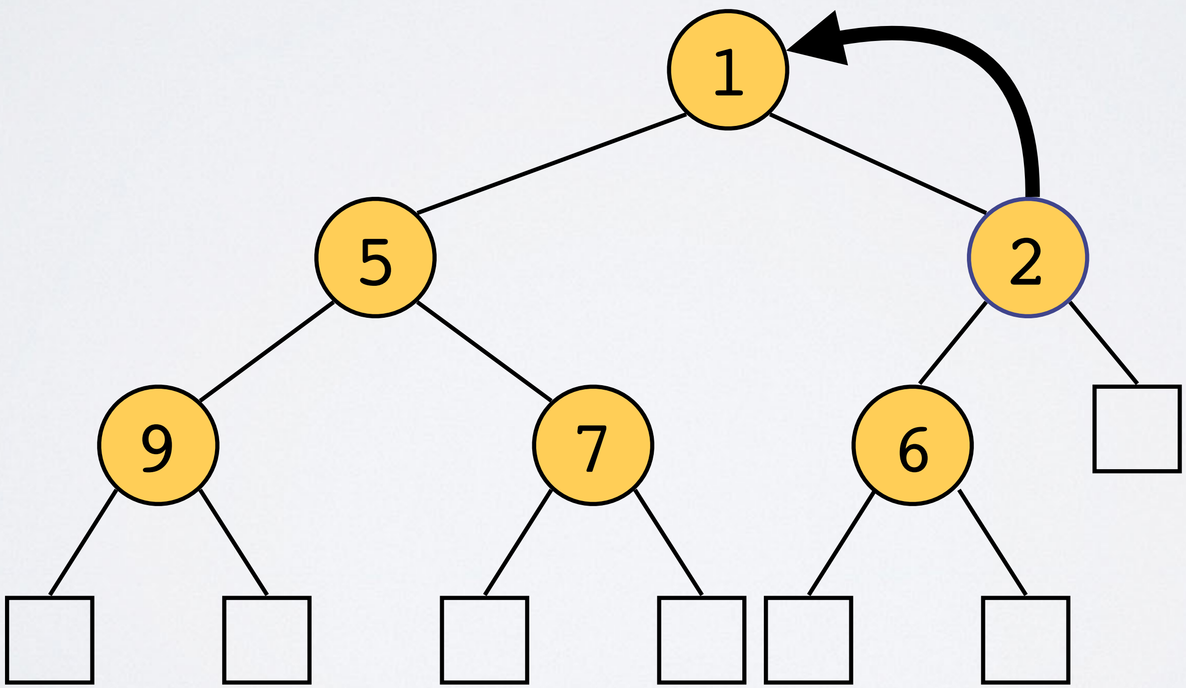
Heap: `upheap`

- Repair heap: swap new element up tree until keys are sorted
- First swap fixes everything below new location
  - since every node below **6**'s old location has to be at least **6**…
  - …they must be at least **1**



Heap: `upheap`

- One more swap since `1≤2`
- Now left-completeness and order are satisfied



Heap: `insert`

Activity #1

2 min

Heap: `insert`

Activity #2

2 min

Heap: `insert`

Activity #1

1 min

Heap: `insert`

Activity #1

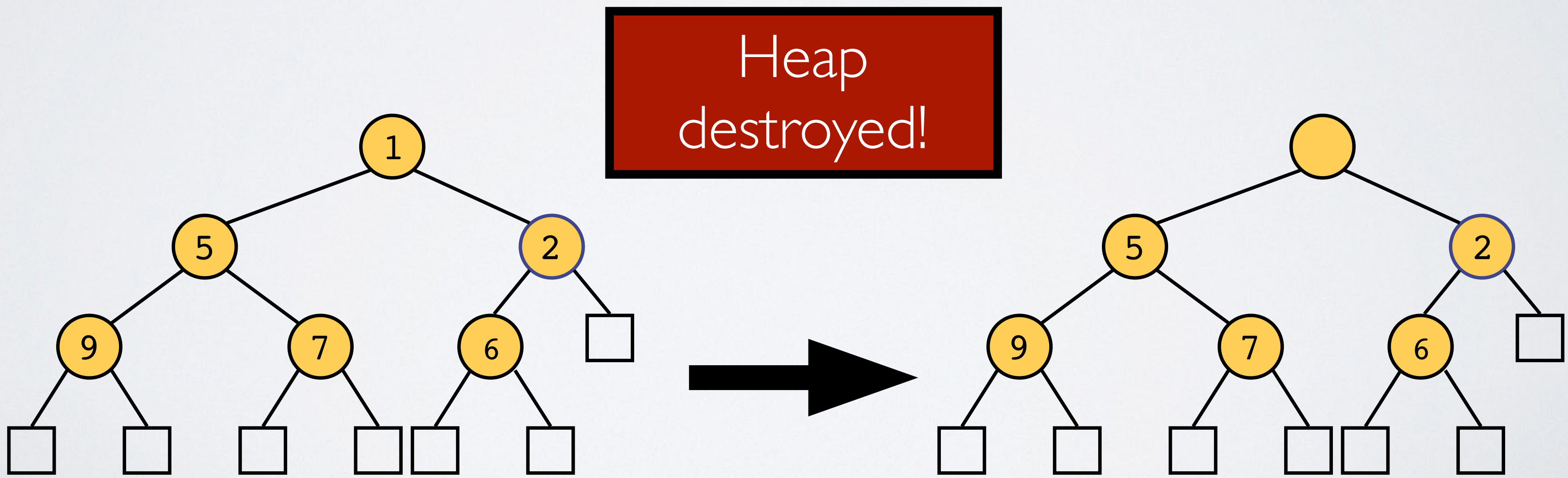
0 min

Heap: `upheap` Summary

- After inserting a key **`k`**, order may be violated
- **`upheap`** restores order by
  - swapping key upward from insertion node
  - terminates when either the root is reached…
  - …or some node whose parent has key less or equal than **`k`**
- Heap insertion has runtime
  - **`O(log n)`**, why?
  - because heap has height **`O(log n)`**
  - perfect binary tree with **`n`** nodes has height **`log(n+1)-1`**

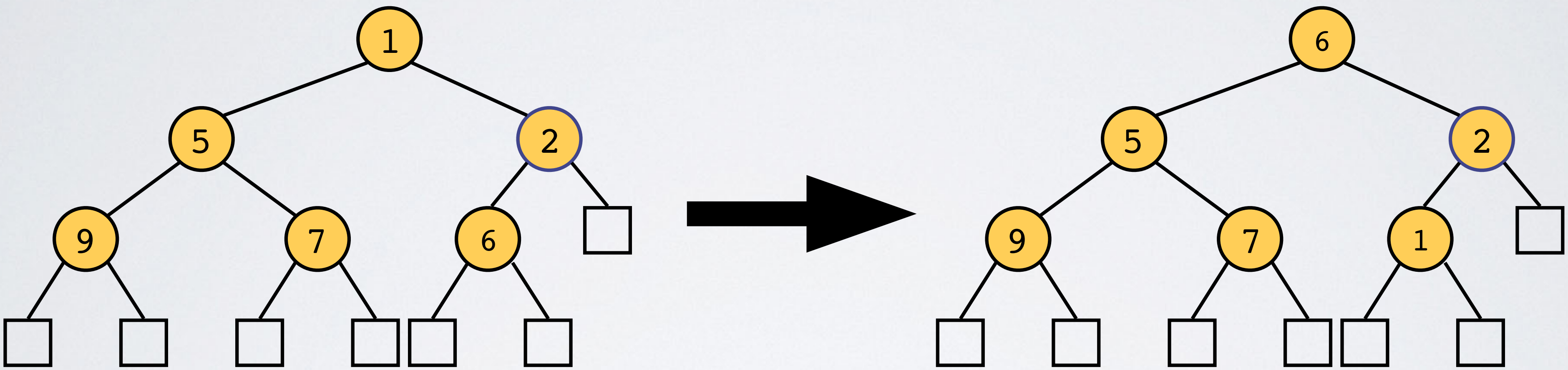
Heap: `removeMin`

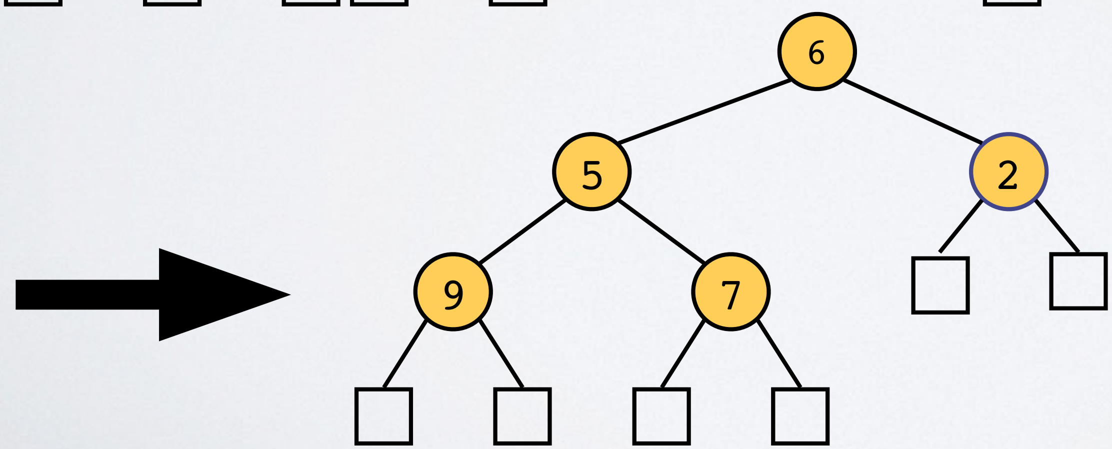
- Remove root
  - because it is always the smallest element
- How can we remove root w/o destroying heap?



Heap: `removeMin`

- Instead swap root with last element & remove it
  - removing last element is easy

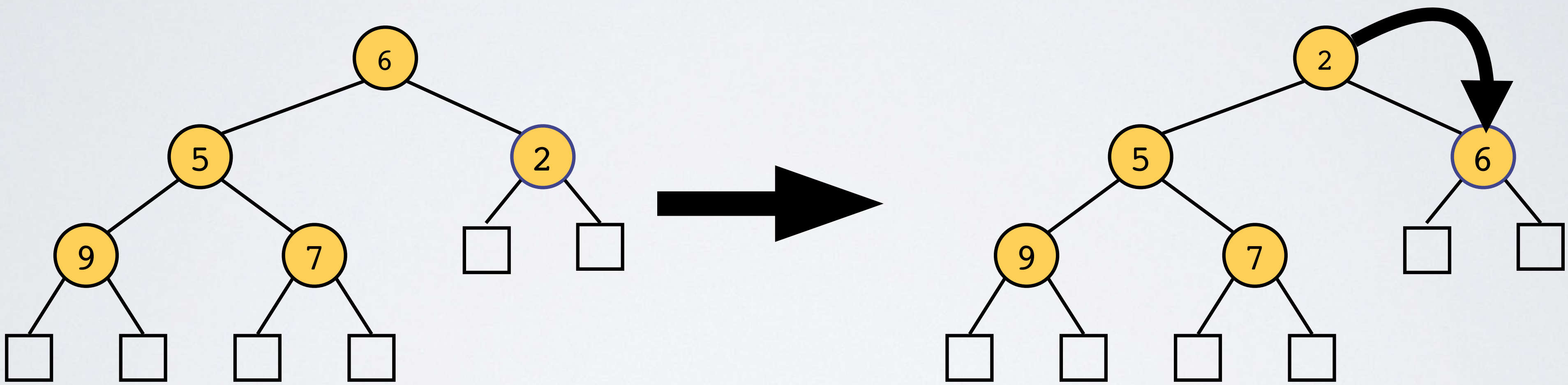




Order destroyed!

Heap: `removeMin`

- Now swap root down as necessary



Heap is in order!

Heap: `downheap` Summary

- **`downheap`** restores order by
  - swapping key downward from the root…
  - …with the smaller of 2 children
  - terminates when either a leaf is reached or
  - …some node whose children has key **`k`** or more
- **`downheap`** has runtime
  - **`O(log n)`**, why?
  - because heap has height **`O(log n)`**

Heap: `removeMin`

Activity #1

2 min

Heap: `removeMin`

Activity #1

2 min

Heap: `removeMin`

Activity #1

1 min

Heap: `removeMin`

Activity #1

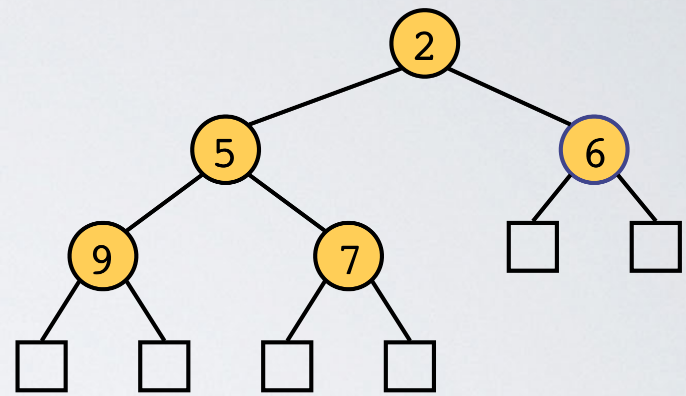
0 min

Summary of Heap

- **`insert(key, value)`**
  - insert value at insertion node
    - insertion node must be kept track of
  - **`upheap`** from insertion node as necessary
- **`removeMin( )`**
  - swap root with last item
  - delete (swapped) last item
  - **`downheap`** from root as necessary

Array-based Heap

- Heap with **`n`** keys can be represented w/ array of size **`n+1`**
- Storing nodes in array
  - Node stored at index **`i`**
    - left child stored at index **`2i`**
    - right child stored at index **`2i+1`**
  - Leaves & edges not stored
  - Cell **`0`** not used
- Operations
  - insert: store new node at index **`n+1`**
  - removeMin: swap w/ index **`n`** and remove

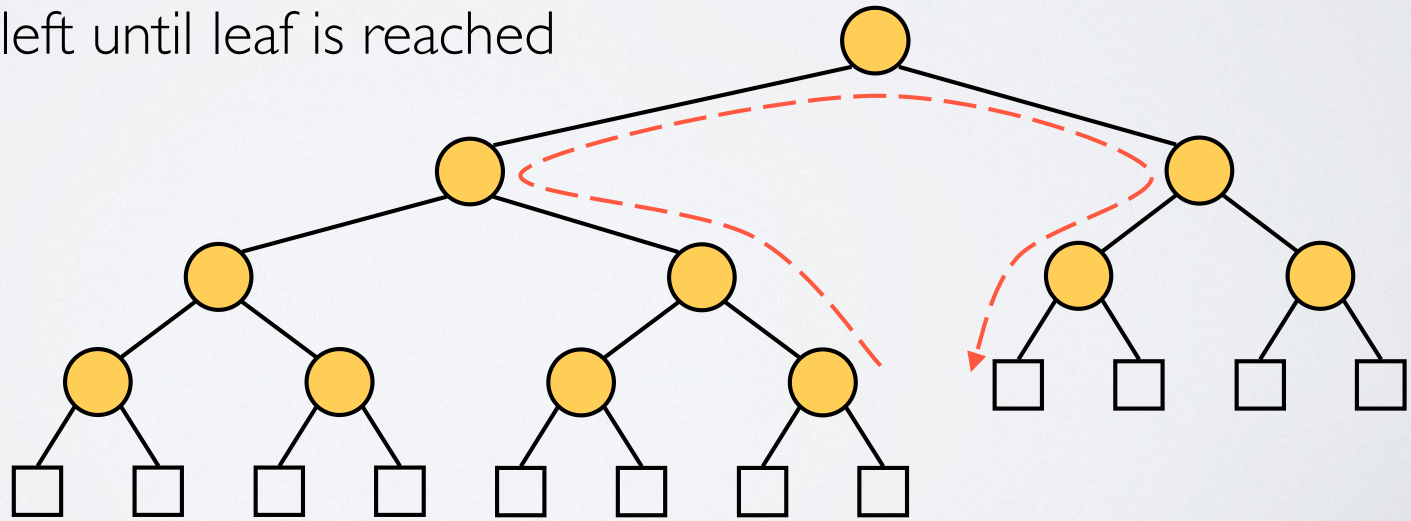


0	1	2	3	4	5
	2	5	6	9	7

Finding Insertion Node

- Can be found in `O(log n)`
- Start at last added node
- Go up until a left child or root is reached
- If left child
  - go to sibling (corresponding right child)
  - then go down left until leaf is reached

Can be done in O(1) time by using additional data structure...need this for project!



Priority Queue Implementation

Implementation	insert	removeMin
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Hash Table	`O(1)`	`O(n)`
Heap	`O(log n)`	`O(log n)`

References

- Slide #4
  - "Queue" in French means tail
  - The picture depicts the tail of a whale
- Slide #7
  - The picture is of a Transformers character named Junkheap which transforms from a waste management garbage truck
- Slide #8
  - The names are characters from the Anime series **Naruto** (https://en.wikipedia.org/wiki/Naruto)
  - The picture is the symbol of the Hidden Leaf Village (where the character Naruto is from)
  - The heap priorities represent the importance of the character in the village