Outline

- Priority Queues
  - Motivation
  - ADT
  - Implementation
- Heaps
  - insert() and upheap()
  - removeMin() and downheap()
Motivation

- Priority queues store items with various priorities
- Priority queues are everywhere
  - Plane departures: some flights have higher priority than others
  - Bandwidth management: real-time traffic like Skype transmitted first
- Student dorm room allocations
- ...
Priority Queue ADT

- Stores key/element pairs
  - key determines position in queue
- `insert(key, element)`:
  - inserts element with key
- `removeMin()`:
  - removes pair w/ smallest key and returns element
Priority Queue Implementations

Activity #1

2 min
Priority Queue Implementations

Activity #1
Priority Queue Implementations
# Priority Queue Implementation

<table>
<thead>
<tr>
<th>Implementation</th>
<th>insert</th>
<th>removeMin</th>
</tr>
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<tbody>
<tr>
<td>Unsorted Array</td>
<td>$O(1)$</td>
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<td>Heap</td>
<td>$O(log \ n)$</td>
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What is a Heap?

- Data structure that implements priority queue
- Heaps can be implemented with
  - Tree
  - Array
- Tree-based heap

```
2
/   \
5   6
/ \   /
9  7 ``
```

Heap Properties

- Binary tree
  - each node has at most 2 children
- Each node has a priority (key)
- Heap has an order
  - min-heap: $n$.parent.key $\leq$ $n$.key
  - max-heap: $n$.parent.key $\geq$ $n$.key
- Left-complete
- Height of $O(\log n)$
Heap Properties

- To implement priority queue
  - insert key/element pair at each node
Heap: **insert**

- Need to keep track of “insertion node”
  - leaf where we will insert new node…
  - …so we can keep heap left-complete
Heap: \textit{insert}

- Ex: \textit{insert}(1)
  - replace insertion node w/ new node

![Heap diagram]

Heap order violated!
Heap: \textit{upheap}

- Repair heap: swap new element up tree until keys are sorted
- First swap fixes everything below new location
  - since every node below 6’s old location has to be at least 6…
  - …they must be at least 1
Heap: upheap

- One more swap since $1 \leq 2$
- Now left-completeness and order are satisfied
Heap: insert

Activity #1

2 min
Heap: insert

Activity #2

2 min
Heap: `insert`

Activity #1

1 min
Heap: insert

Activity #1
Heap: **upheap** Summary

- After inserting a key $k$, order may be violated
- **upheap** restores order by
  - swapping key upward from insertion node
  - terminates when either the root is reached...
  - …or some node whose parent has key less or equal than $k$
- Heap insertion has runtime
  - $O(\log n)$, why?
  - because heap has height $O(\log n)$
  - perfect binary tree with $n$ nodes has height $\log(n+1) - 1$
Heap: `removeMin`

- Remove root
  - because it is always the smallest element
- How can we remove root w/o destroying heap?
Heap: `removeMin`

- Instead swap root with last element & remove it
  - removing last element is easy
Heap: \texttt{removeMin}

- Now swap root down as necessary

Heap is in order!
Heap: downheap Summary

- **downheap** restores order by
  - swapping key downward from the root...
  - …with the smaller of 2 children
  - terminates when either a leaf is reached or
  - …some node whose children has key $k$ or more

- **downheap** has runtime
  - $O(\log n)$, why?
  - because heap has height $O(\log n)$
Heap: \texttt{removeMin}

2 min

Activity #1
Heap: \texttt{removeMin}

\textbf{Activity \#1}

2 min
Heap: removeMin

Activity #1
Heap: `removeMin`
Summary of Heap

- **insert**(key, value)
  - insert value at insertion node
    - insertion node must be kept track of
  - **upheap** from insertion node as necessary

- **removeMin**( )
  - swap root with last item
  - delete (swapped) last item
  - **downheap** from root as necessary
Array-based Heap

- Heap with \( n \) keys can be represented w/ array of size \( n+1 \)
- Storing nodes in array
  - Node stored at index \( i \)
    - left child stored at index \( 2i \)
    - right child stored at index \( 2i+1 \)
  - Leaves & edges not stored
  - Cell 0 not used
- Operations
  - insert: store new node at index \( n+1 \)
  - removeMin: swap w/ index \( n \) and remove
Finding Insertion Node

- Can be found in $O(\log n)$
- Start at last added node
- Go up until a left child or root is reached
- If left child
  - go to sibling (corresponding right child)
  - then go down left until leaf is reached

Can be done in $O(1)$ time by using additional data structure...need this for project!
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References

‣ Slide #4
  ‣ “Queue” in French means tail
  ‣ The picture depicts the tail of a whale

‣ Slide #7
  ‣ The picture is of a Transformers character named Junkheap which transforms from a waste management garbage truck

‣ Slide #8
  ‣ The names are characters from the Anime series Naruto (https://en.wikipedia.org/wiki/Naruto)
  ‣ The picture is the symbol of the Hidden Leaf Village (where the character Naruto is from)
  ‣ The heap priorities represent the importance of the character in the village