Structure of an Induction Proof – CS16 Spring 2019

The left side of the handout is a step-by-step stencil for an inductive proof. The right side walks through these steps given an example.

To prove: write out statement here…

We’ll prove this by induction. Let \( P(n) \) be the statement:

\[ \text{fill in predicate here…} \]

Base case: We’ll first prove that \( P(1) \) is true.

write out \( P(1) \) here, and give an explanation of why it’s true.

\[ (1 + a)^1 \geq 1 + a 
\]

\( P(1) \) is true because it simplifies to \( 1 + a \geq 1 + a \)

Inductive step: Assume \( P(k) \) is true for some positive integer \( k \):

write out \( P(k) \) here

\[ P(k) : (1 + a)^k \geq 1 + ak \]

We’ll show that \( P(k) \) implies \( P(k + 1) \):

write out \( P(k + 1) \) here.

\[ P(k + 1) : (1 + a)^{k+1} \geq 1 + a(k+1) \]

Start from \( P(k) \) and argue the truth of \( P(k + 1) \).

We’ve assumed:

\[ (1 + a)^k \geq 1 + ak \]

Multiplying both sides by \( 1 + a \), we get:

\[ (1 + a)^{k}(1 + a) \geq (1 + ak)(1 + a) \]

\[ (1 + a)^{k+1} \geq 1 + a + ak + a^2k \]

Refactoring the right hand side, we get:

\[ (1 + a)^{k+1} \geq 1 + a(k + 1) + a^2k \]

If this is true, then the following is also true, because \( a^2k \) is positive

\[ (1 + a)^{k+1} \geq 1 + a(k + 1) \]

which is exactly the statement \( P(k + 1) \), which we promised to prove.

Since \( P(1) \) is true, and \( P(k) \rightarrow P(k + 1) \), by induction

\( P(n) \) is true for all integers \( n \geq 1 \)

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