Structure of an Induction Proof – CS16 Spring 2020

To prove: write out statement here...  
To prove: if \( a > -1 \), then for every integer \( n \geq 1 \), \( (1 + a)^n \geq 1 + an \)

We’ll prove this by induction. Let \( P(n) \) be the statement: 
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\[
\text{fill in predicate here...} \quad (1 + a)^n \geq 1 + an
\]

Base case: We’ll first prove that \( P(1) \) is true. 
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write out \( P(1) \) here, and give an explanation of why it’s true.

\[
(1 + a)^1 \geq 1 + a \cdot 1
\]

\( P(1) \) is true because it simplifies to \( 1 + a \geq 1 + a \)

Inductive step: Assume \( P(k) \) is true for some positive integer \( k \):
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\[
P(k) \quad : (1 + a)^k \geq 1 + ak
\]

We’ll show that \( P(k) \) implies \( P(k + 1) \):
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\[
P(k+1) \quad : (1 + a)^{k+1} \geq 1 + a(k + 1)
\]

Start from \( P(k) \) and argue the truth of \( P(k + 1) \).
We’ve assumed:

\[
(1 + a)^k \geq 1 + ak
\]

Multiplying both sides by \( 1 + a \), we get:

\[
(1 + a)^{k+1} \geq (1 + ak)(1 + a)
\]

\[
(1 + a)^{k+1} \geq 1 + a + ak + a^2k
\]

Refactoring the right hand side, we get:

\[
(1 + a)^{k+1} \geq 1 + a(k + 1) + a^2k
\]

If this is true, then the following is also true, because \( a^2k \) is positive

\[
(1 + a)^{k+1} \geq 1 + a(k + 1)
\]

which is exactly the statement \( P(k + 1) \), which we promised to prove.

Since \( P(1) \) is true, and \( P(k) \rightarrow P(k + 1) \), by induction \( P(n) \) is true for all integers \( n \geq 1 \)
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