Structure of an Induction Proof – CS16 Summer 2021

The left side of the handout is a step-by-step stencil for an inductive proof. The right side walks through these steps given an example.

To prove: *write out statement here...*

We'll prove this by induction. Let $P(n)$ be the statement:

fill in predicate here...

Base case: We'll first prove that $P(1)$ is true.
write out P(1) here, and give an explanation of why it's true.

Inductive hypothesis:

Assume $P(k)$ is true for some positive integer k.

write out $P(k)$ here

Inductive step:
We'll show that $P(k)$ implies $P(k+1)$:

write out $P(k+1)$ here.

Start from $P(k)$ and argue the truth of $P(k+1)$.

Since $P(1)$ is true, and $P(k) \rightarrow P(k+1)$, by induction $P(n)$ is true for all integers $n \geq 1$

To prove: if $a > -1$, then for every integer $n \geq 1$, $(1+a)^n \geq 1 + an$

We'll prove this by induction. Let $P(n)$ be the statement:

$$(1+a)^n \geq 1 + an$$

Base case: We'll first prove that $P(1)$ is true.

$$(1+a)^1 \geq 1 + a \cdot 1$$

$P(1)$ is true because it simplifies to $1 + a \geq 1 + a$

Inductive hypothesis:

Assume $P(k)$ is true for some positive integer k.

$$P(k) : (1+a)^k \geq 1 + ak$$

Inductive step:
We'll show that $P(k)$ implies $P(k+1)$:

$$P(k+1) : (1+a)^{k+1} \geq 1 + a(k+1)$$

We've assumed:

$$(1+a)^k \geq 1 + ak$$

Multiplying both sides by $1 + a$, we get:

$$(1+a)^k(1+a) \geq (1+ak)(1+a)$$

$$(1+a)^{k+1} \geq 1 + a + ak + a^2k$$

Refactoring the right hand side, we get:

$$(1+a)^{k+1} \geq 1 + a(k+1) + a^2k$$

If this is true, then the following is also true, because a^2k is positive

$$(1+a)^{k+1} \geq 1 + a(k+1)$$

which is exactly the statement $P(k+1)$, which we promised to prove.

Since $P(1)$ is true, and $P(k) \rightarrow P(k+1)$, by induction $P(n)$ is true for all integers $n \geq 1$