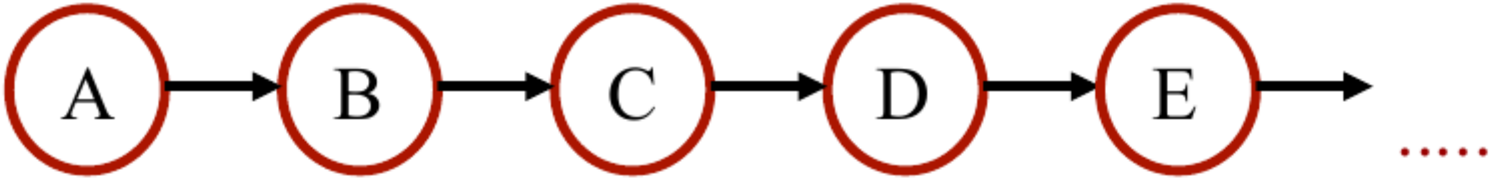
Overview

- Definitions, Terminology, and Properties
- Binary Trees
- Search Trees: Improving Search Speed
- Traversing Binary Search Trees

Searching in a Linked List (1/3)

- Searching for element in `LinkedList` involves pointer-chasing and checking consecutive `Node`s to find it (or not)
    - It is sequential access
    - O(N) – can stop sooner for element not found if list is sorted
- Finding i'th element in `array` or `ArrayList` is random access (which means O(1)), but searching for particular element (even with index) remains sequential O(N)
- Even though `NodeLists` support indexing (dictated by Java's list interface), finding the i'th element is also done (under the hood) by pointer-chasing and hence is O(N)

Searching in a Linked List (2/3)



- Searching for **E**:
  - start at **A**, beginning of list
  - but **A** is not **E** – continue to node **B**
  - but **B** is not **E**, continue to node **C** (and so on…)
  - till… **E** is **E**, found it!
  - or it isn't in list – exit on null (unsorted) or first element greater (sorted)

Searching in a Linked List (3/3)

- For N elements, search time is O(N)
  - unsorted: sequentially check every node in list till element ("search key") being searched for is found, or end of list is reached
    - if in list, for a uniform distribution of keys, average search time is N/2
    - if not in list, it is N
  - sorted: average search time is N/2 if found, N/2 if not found (the win!)
  - we ignore issue of duplicates
- No efficient way to access N^{th} node in list
- Insert and remove similarly have average search time of N/2 to find the right place
- Is there another data structure that provides faster search time and still fast updating of the data structure?

Binary Search (1/5)

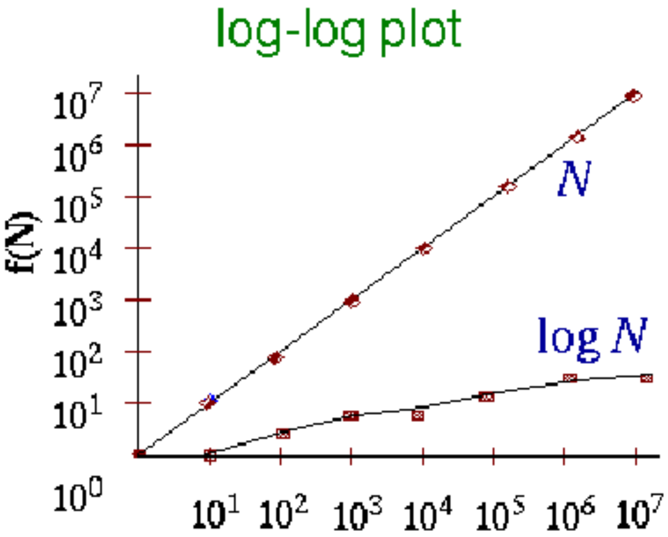
- Worst case for searching sorted linked list is checking every element i.e., sequential access
- We can do better with a sorted array which allows random access at any index
- Let's **demo** binary search ("bisection") for a sorted list of numbers
- Website: http://www.cs.armstrong.edu/liang/animation/web/BinarySearch.html

Binary Search (2/5)

- For N elements, search time is $O(\log_2 N)$ (since we reduce number of elements to search by half each time), very efficient!
  - start in the middle
  - keep bisecting the array by halving the index, deciding which half interval the search key lies in, until we land on that key or can't subdivide further (not in array)

Binary Search (3/5)

- $\log_2 N$ is considerably smaller than N, especially for large N. So doing things in time proportional to $\log_2 N$ is better than doing them in proportion to N!



N	log(N)
1	0
10	3
100	7
1,000	10
10,000	13
100,000	17
1,000,000	20
10,000,000	23
100,000,000	27
1,000,000,000	30

Binary Search (4/5)

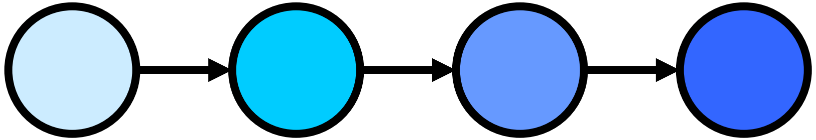
- A sorted array can be searched quickly using bisection because arrays are indexed
- Java `ArrayLists` are indexed too, so they share this advantage! But inserting and removing from `ArrayLists` is slow!
- Inserting into or deleting from middle of `ArrayList` causes all successor elements to be shifted over to make room. For arrays, you manage this yourself. Both have same worst-case run time – O(N)
- Advantage of linked lists is insert/remove by manipulating pointer chain is faster [O(1)] than shifting elements [O(N)], but search can’t be done with bisection ☹, a real downside if search is done frequently

Binary Search (5/5)

- Is there a data structure/Abstract Data Type that provides both search speed of sorted arrays and `ArrayList`s and insertion/deletion efficiency of linked lists?
- Yes, indeed! `Tree`s!

Trees vs Linked Lists (1/2)

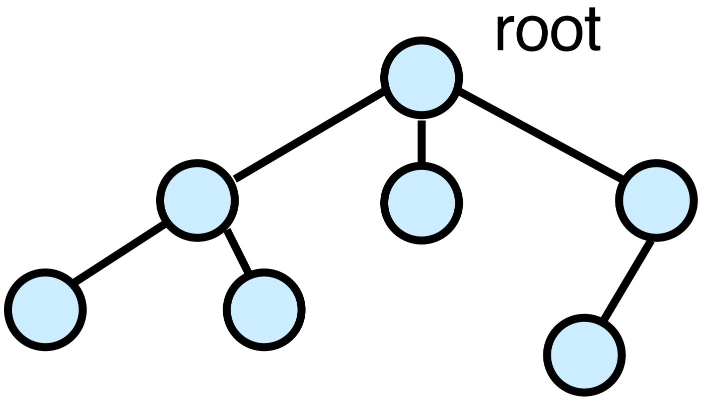
- Singly linked list – collection of nodes where each node references only one neighbor, the node's successor:



Trees vs Linked Lists (2/2)

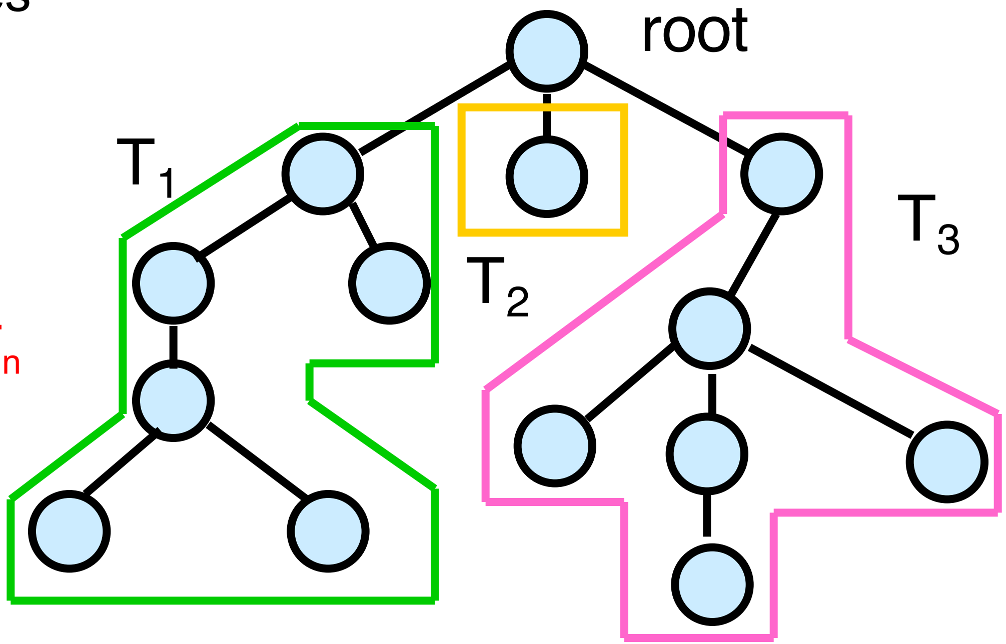
- Tree – also collection of nodes, but each node may reference multiple successors/children

- Trees can be used to model a hierarchical organization of data

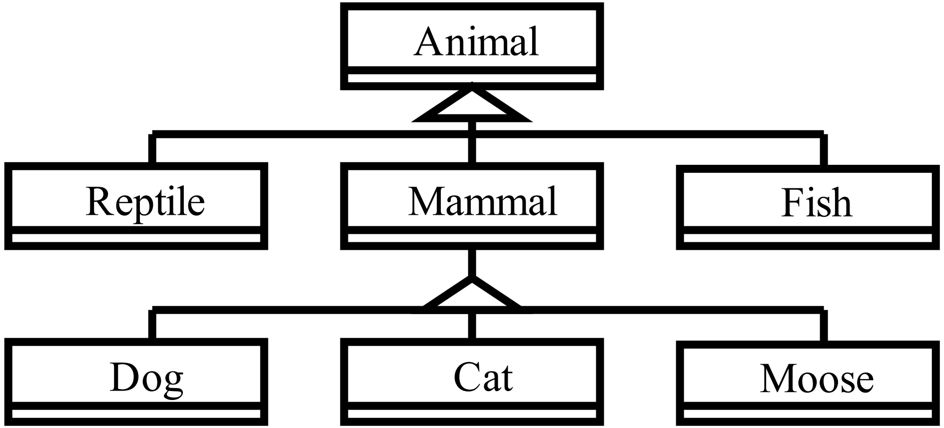


Technical definition of a Tree

- Finite set, T, of one or more nodes such that:
  - T has one designated root node
  - remaining nodes partitioned into disjoint sets: T_1, T_2, … T_n
  - each T_i is also a tree, called subtree of T
- Look at the image on the right-where have we seen seen such hierarchies like this before?



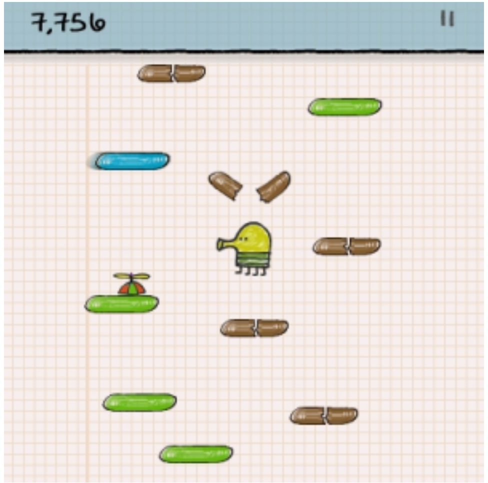
Inheritance Hierarchies as Trees

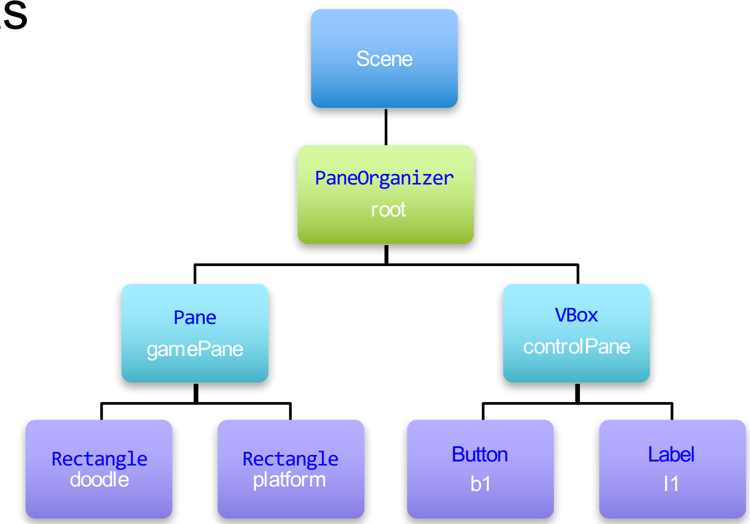


- Higher in inheritance hierarchy, more generic
  - **Animal** is most generic
- Lower in inheritance hierarchy, more specific
  - **Dog** is more specific than **Mammal**, which in turn is more specific than **Animal**

Graphical Containment Hierarchies as Trees

- Levels of containment of **GUI** components

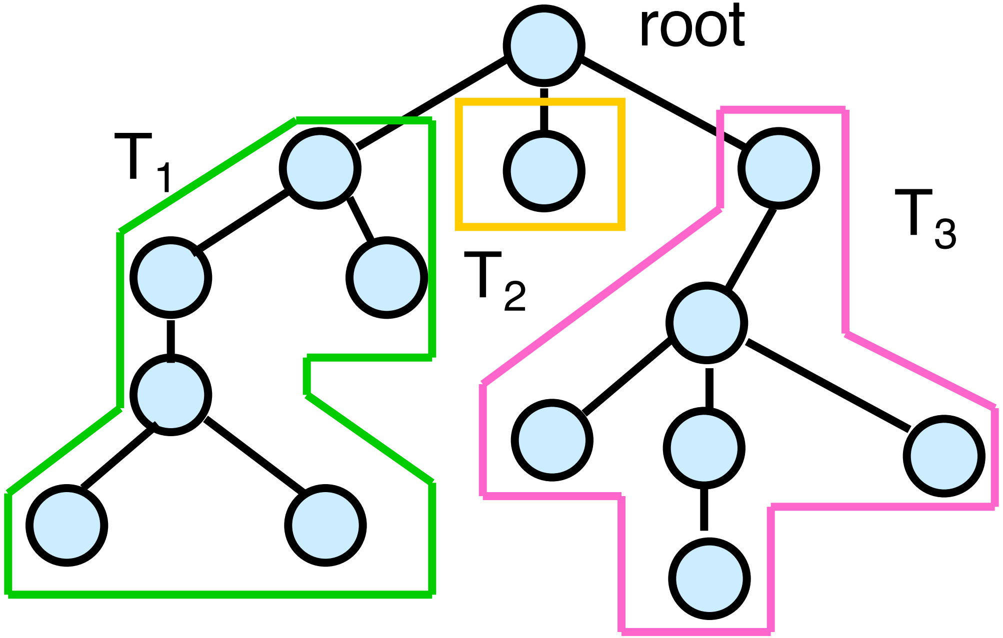




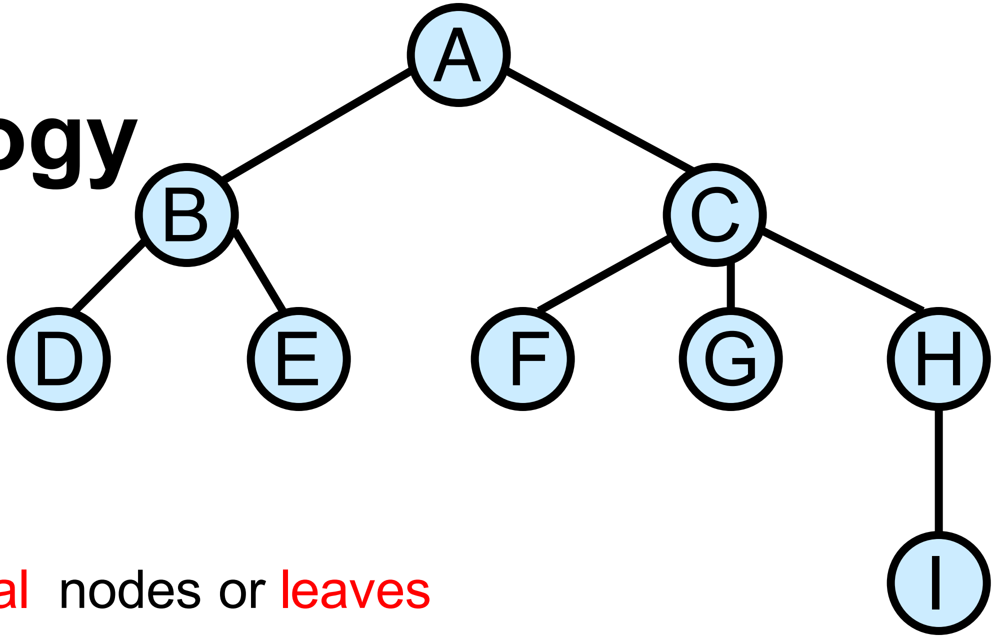
- Higher levels contain more components
- Lower levels contained by all above them
    - **Panes** contained by **Root Pane** of **PaneOrganizer**, which is contained by **Scene**

Tree Structure

- Note that the tree structure has meaning
  - Any subtree of T, T_i ,is also a tree with specific values
- Can be useful to only examine specific subtrees of T



Tree Terminology

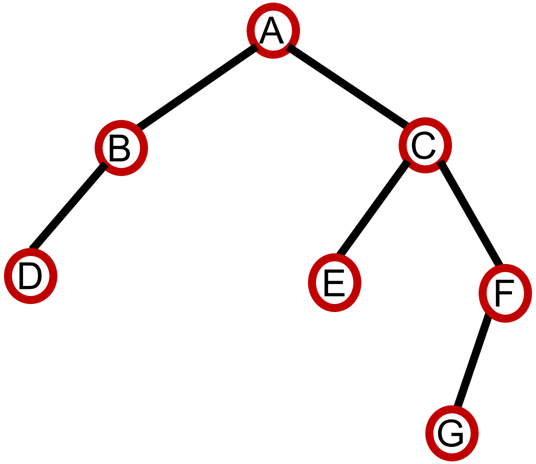


- A is the root node
- B is the parent of D and E
- D and E are children of B
- (C ---- F) is an edge
- D, E, F, G, and I are external nodes or leaves (i.e., nodes with no children)
- A, B, C, and H are internal nodes
- depth (level) of E is 2 (number of edges to root)
- height of the tree is 3 (max number of edges in path from root)
- degree of node B is 2 (number of children)

Binary Trees

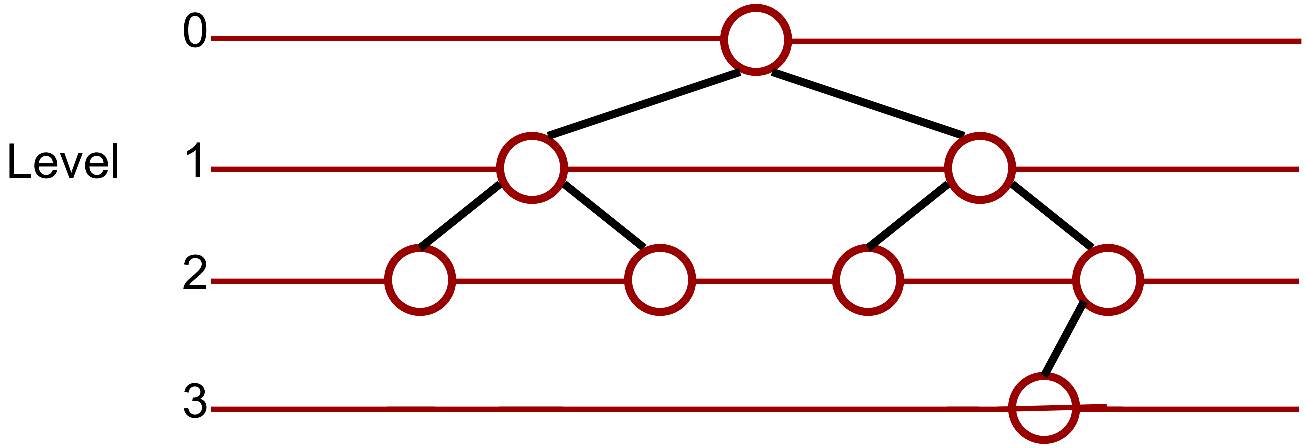
- Each internal node has a maximum of 2 successors, called children
  - So, each internal node has degree 2 at most
- Recursive definition of binary tree: A binary tree is either an:
  - External node (leaf), or
  - Internal node (root) with two binary trees as children (left subtree and right subtree)
  - Empty tree (represented by a null pointer)

Note: These nodes are similar to the linked list nodes, with one data and two child pointers

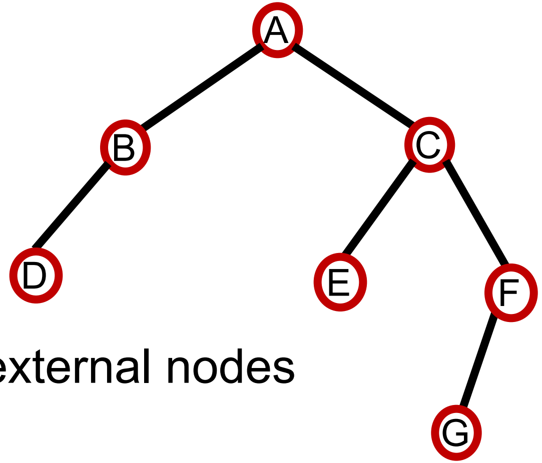


Properties of Binary Trees (1/2)

- A Binary tree is full when each node has exactly zero or two children
- Binary tree is perfect when for every level i, there are 2^i nodes (i.e., each level contains a complete set of nodes)
  - Thus, adding anything to the tree would increase its height



Clicker Question



Which of the following are true about internal nodes and external nodes for binary trees?

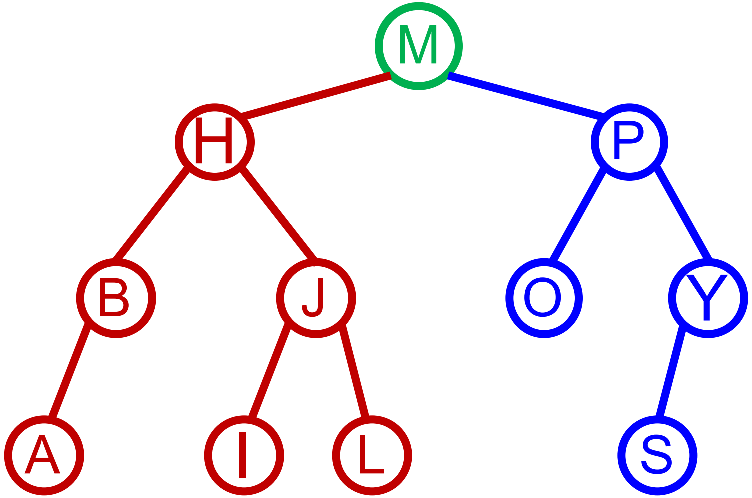
A. Internal nodes are nodes located between the root of the tree and leaves
B. External nodes cannot be leaves of a tree
C. The root node is often an external node
D. Internal nodes must have one or two children, while external nodes have none

Properties of Binary Trees (2/2)

- In a full Binary Tree: (# leaf nodes) =(# internal nodes) + 1
- In a perfect Binary Tree: (# nodes at level i) <= 2^i
- In a perfect Binary Tree: (# leaf nodes) <= $2^{(height)}$
- In a perfect Binary Tree(height) >= $\log_2$(# nodes) - 1

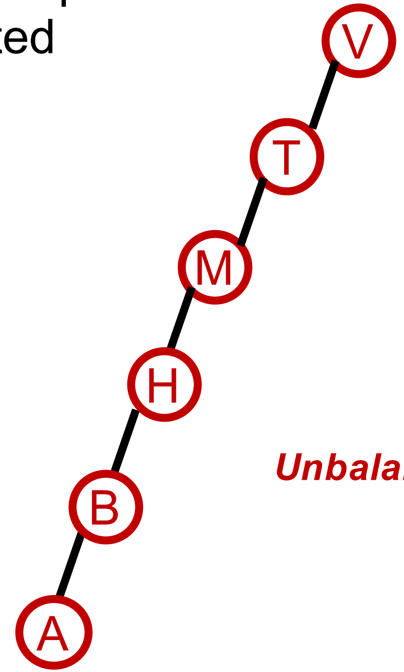
Binary Search Tree a.k.a BST (1/2)

- Binary search tree stores keys in its nodes such that, for every node, keys in left subtree are smaller, and keys in right subtree are larger

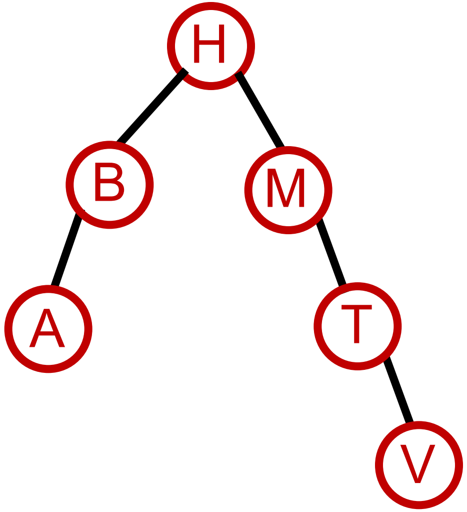


Binary Search Tree (2/2)

- Below is also binary search tree but much less balanced. Gee, it looks like a linked list!
- The shape of the trees is determined by the order in which elements are inserted



Unbalanced



Balanced

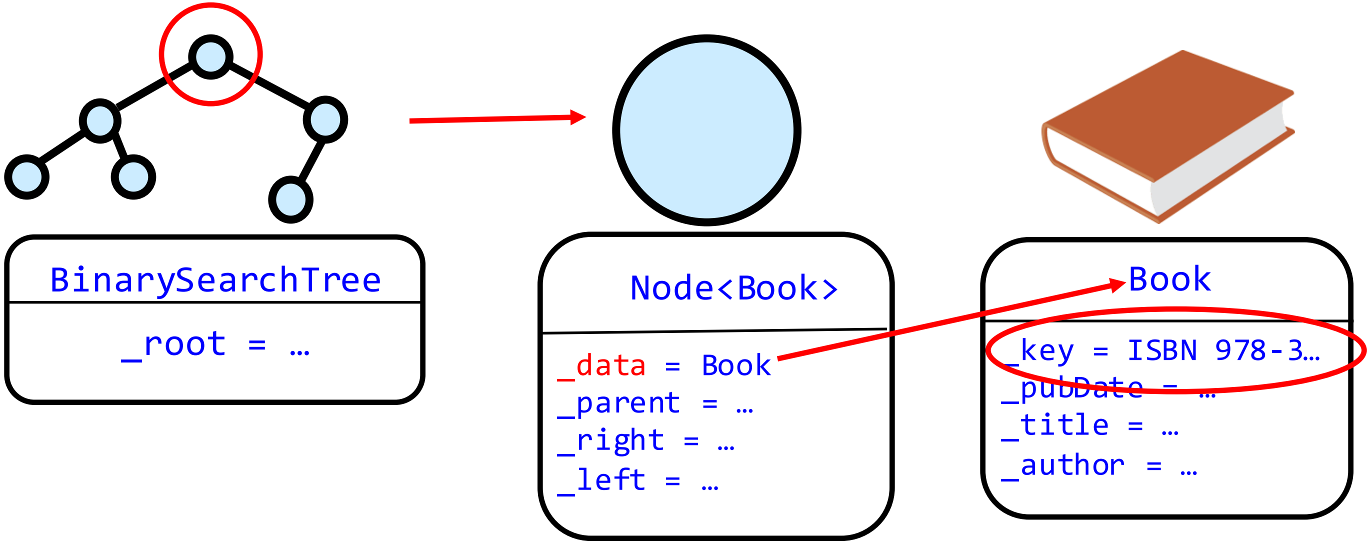
Binary Search Tree Class (1/3)

- What do binary search trees know how to do?
  - much the same as sorted linked lists: *insert*, *remove*, *size*
  - BSTs also have a special search method, since searching is more complicated than simply iterating through its nodes

- Let's see what an implementation of a binary search tree class would look like…
  - you'll learn more about implementing classes of data structures in CS16!

Nodes, data, and keys

- _data is a composite that can contain many properties, one of which is a key that Nodes are sorted by (here, ISBN number)



Binary Search Tree Class (2/3)

```
public class BinarySearchTree<Type extends
  Comparable<Type>> {

    Node<Type> _root;
    public BinarySearchTree(Type data){
        //Root of the tree
        _root = new Node(data, null);
    }

    public void insert(Type newData) {
      // . . .
     }
```

```
    //class continued
    public void remove(Type dataToRemove) {
     // . . .
    }
    public Node<Type> search(Type dataToFind) {
     // ...
    }
    public int size() {
     // ...
    }

} //end of class
```

Binary Search Tree Class (3/3)

- Our implementations of `Linked Lists`, `Stacks` and `Queues` are "smart" data structures that chain "dumb" nodes together.
  - the lists did all the work by maintaining prev and curr pointers and did the operations to search for, insert and remove information
- Now we can have a "dumb" tree with "smart" nodes that will delegate using recursion
  - tree will delegate action (such as searching, inserting, etc.) to its root, which will then delegate to its appropriate child, and so on.
  - different from linear linked lists, stacks, queues: create specialized Node class that knows about its data, parent, and children

Binary Search Tree: Node Class (1/3)

- "Smart" Node includes the following methods:

```
//pass in entire data item, containing key, not just key, so compareTo() will work
public Node<Type> search(Type dataToFind);
public Node<Type> insert(Type newData);

/*Remove deletes Node pointing to dataToRemove, which contains key; removing Node
also will remove the data instance unless there's another reference to it*/
public Node<Type> remove(Type dataToRemove);
public Node<Type> swapData(); //swaps the data in two different nodes
```

- Plus setters and getters of instance variables, defined in the next slides …

Binary Search Tree: Node Class (2/3)

- Nodes have a maximum of two non-null children that hold data implementing Comparable<Type>
  - Four instance variables: _data, _parent, _left, and _right, with each having a get and set method.
  - _data represents the data that Node stores. It also contains the key attribute that Nodes are sorted by – we'll use a tree that stores books
  - _parent represents the direct parent (another Node) of Node – only used in remove method
  - _left represents Node's left child and contains a subtree, all of whose data is less than Node's data
  - _right represents Node's right child and contains a subtree, all of whose data is greater than Node's data
  - Arbitrarily select which child should contain data equal to Node's data

Binary Search Tree: Node Class (3/3)

```
public class Node<Type extends Comparable<Type>> {
    private Type _data;
    private Type _parent;
    private Node<Type> _left;
    private Node<Type> _right;
    public Node(Type data, Node<Type> parent){//construct a leaf node
        _data = data;
        _parent = parent;
        //children start as null – their values are set when children nodes are created
        _left = null;
        _right = null;
    }
   //Will define other methods in next slides…
}
```

Aside: What about leaf Nodes?

- A leaf node has no descendants
  - We want to use a similar design to MyLinkedList, using null pointer
- Every Node starts as a leaf, as _left and _right start as null:

```
public Node(Type data, Node<Type> parent){ //constructor from previous slide
        _data = data;
        _parent = parent;
        _left = null;
        _right = null;
}
```

- We can simply reassign values of _left and _right using mutator methods when we insert new Nodes into the tree

Smart Node Approach

- `BinarySearchTree` is dumb so it delegates to root, which in turn will delegate recursively to its left or right child, as appropriate

```
// search method for entire BinarySearchTree:
public Node<Type> search(dataToFind) {
    return _root.search(dataToFind);
}
```

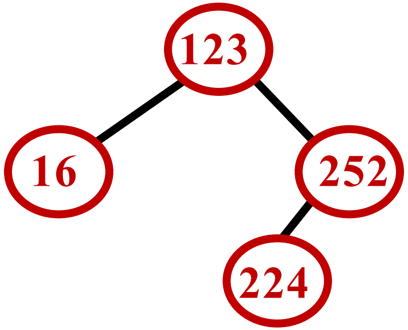
- Smart node approach makes our code clean, simple and elegant
  - non-recursive method is much messier, involving explicit bookkeeping of which node in the tree we are currently processing
    - we used the non-recursive method for sorted linked lists, but trees are more complicated, and recursion is easier

Recursion – Turtles all the Way Down

- “A well-known scientist (some say it was Bertrand Russell) once gave a public lecture on astronomy. He described how the earth orbits around the sun and how the sun, in turn, orbits around the center of a vast collection of stars called our galaxy. At the end of the lecture, a little old lady at the back of the room got up and said: "What you have told us is rubbish. The world is really a flat plate supported on the back of a giant tortoise." The scientist gave a superior smile before replying, "What is the tortoise standing on?" "You're very clever, young man, very clever," said the old lady. "But it's turtles all the way down!"”

  -Stephen Hawking, *A Brief History of Time* (1988)

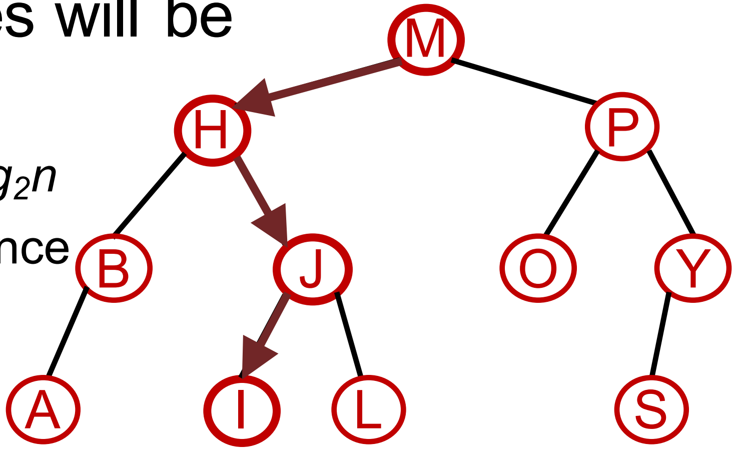
Searching Simulation



- What if we want to know if **224** is in Tree?
- Tree says "Hey Root! Ya got **224**?"
- **123** says: "Let's see. I'm not **224**. But if **224** is in tree, it would be to my right. I'll ask my right child and return its answer."
- **252** says: "I'm not **224**. I better ask my left child and return its answer."
- **224** says: "**224**? That's me! Hey, caller **(252)** here's your answer." (returning node indicates that query is in tree)
- **252** says: "Hey, caller **(123)**! Here's your answer."
- **123** says: "Hey, Tree! Here's your answer."

Searching a Binary Tree Recursively

- Search path: start with root **M** and choose path to **I** (for a reasonably balanced tree, M will be more or less "in the middle", and left and right subtrees will be roughly the same size)
  - The height of the tree with n nodes is $log_2 n$
  - At most, we visit each level of the tree once
  - So, searching is $O(\log_2 N)$



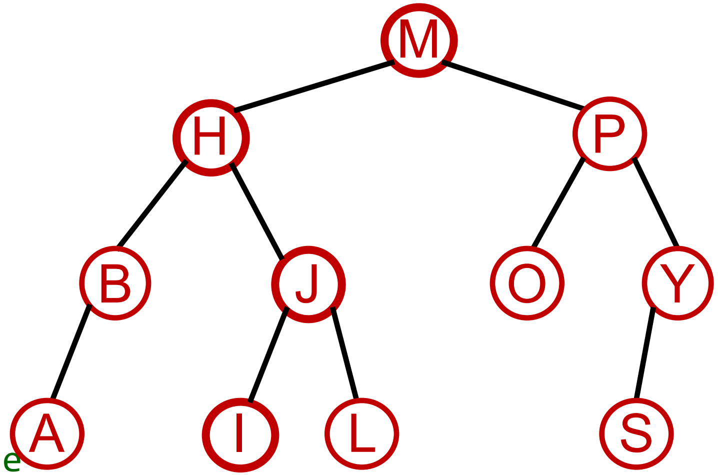
Clicker Question

What's the runtime of (recursive) search in a BST and why?

A. O(n) – because you only iterate once
B. O(2n) – because you go through the left and right children
C. O(n/2) – because you incorporate the idea of “bisection” to mean half the nodes
D. O($\log_2$n) - because you incorporate the idea of “bisection” to eliminate half the number of nodes to search at each recursion
E. O(n^2) – because recursion makes your runtime quadratic

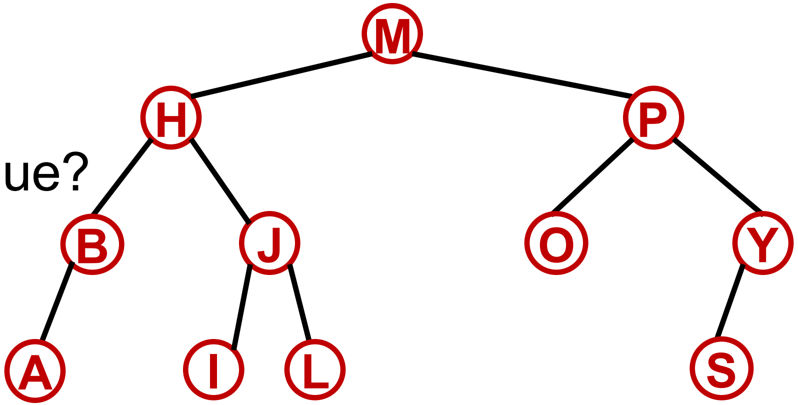
Searching a Binary Tree Recursively

```
public Node<Type> search(Type dataToFind){
    //if _data is the thing we're searching for
    if(_data.compareTo(dataToFind) == 0){
        return this;
    //if _data > dataToFind, can only be in left tree
    } else if(_data.compareTo(dataToFind) > 0){
        if(_left!= null){
            return _left.search(dataToFind);
        }
    //if _data < dataToFind, can only be in right tree
    } else {
        if(_right != null){
            return _right.search(dataToFind);
        }
    }
    //Only gets here if dataToFind isn't in tree, otherwise would've returned sooner
    return null;
}
```



Binary Search: Nearest Value

- What if we wanted to find the closest possible value, rather than the exact value?
  - search(H) would return H
  - search(D) would return B
- Can use binary search with one modification
  - instead of immediately returning the element if you find it, have a variable that keeps track of the closest neighbor



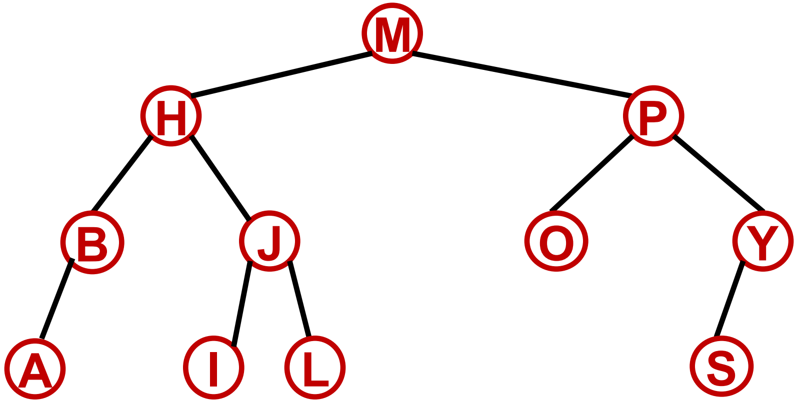
Nearest Value Pseudocode

```
search (type dataToFind, Node currNode): //method in BinaryTree class
     if currNode is null: //base case
         return _closestNode //initialized elsewhere in BinaryTree class
    //see how close current data is from target data
    //getDistanceFrom returns an integer distance between nodes, but type isn't always int
     distToTarget = currNode.getDistanceFrom(dataToFind)
     //if this is closer than our current assumption of closest node, update
     if distToTaget < _closestDist:
          _closestDist = distToTarget
          _closestNode = currNode
//if the data is equal to our target data, return immediately (can't get closer)
     if currNode.data is equal to dataToFind:
          return _closestNode
     else if currNode.data < dataToFind: //otherwise, recurse
          return this.search(dataToFind, currNode.left)
     else:
          return this.search(dataToFind, currNode.right)
```

Note: this pseudocode doesn't use a smart Node class.

Binary Search: Nearest Values

- What if we wanted to find the closest x possible values, rather than just the closest value?
  - search(H, 1) would return {H}
  - search(D, 2) would return {A, B}
- Can use method similar to binary search
  - Find closest neighbors to point, then find closest neighbors to immediate neighbors
  - Won't go into detail, but this is a powerful advantage of the BST

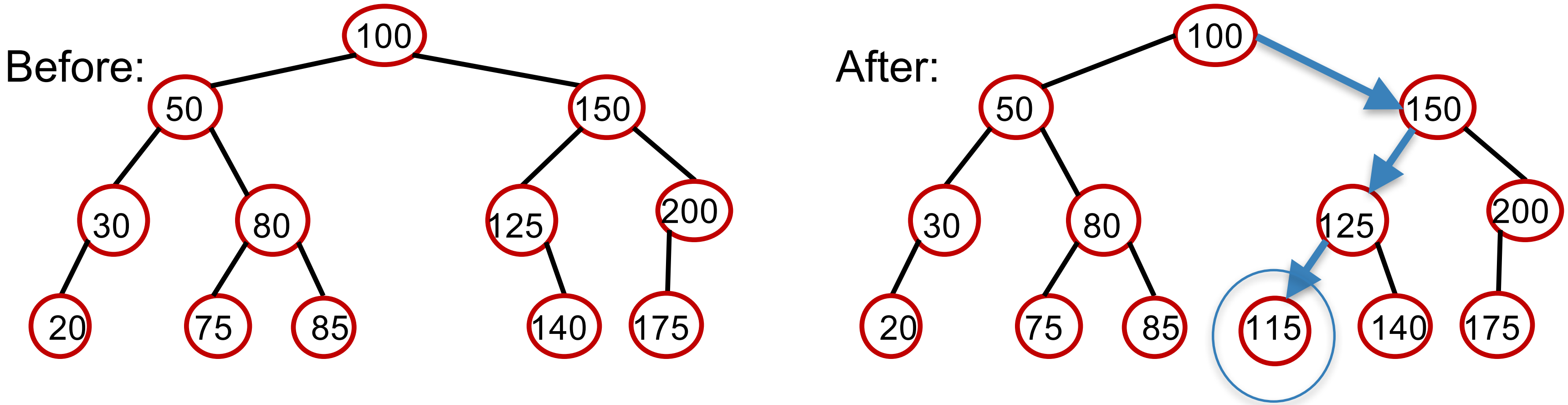


Insertion into a BST(1/2)

- Search BST starting at root until we find where the data to insert belongs
  - Insert data when we reach a `Node` whose appropriate child is `null`
- We make a new `Node`, set the new `Node`'s `_data` to the data to insert, and set the parent's child reference to this `Node`.
- Runtime is $O(\log_2 N)$, yay!
  - $O(\log_2 N)$ to search the tree to find the place to insert
  - Constant time operations to make new `Node`

Insertion into a BST(2/2)

- Example: Insert **115**



Insertion Code in BST

- Again, we use a "Smart Node" approach and delegate

```
public void insert(Type newData) {
    //if tree is empty, make first node. No traversal necessary!
    if(_root == null){
        _root = new Node(newData, null);
    } else{
        _root.insert(newData); // delegate
    }
}
```

Insertion Code in Node

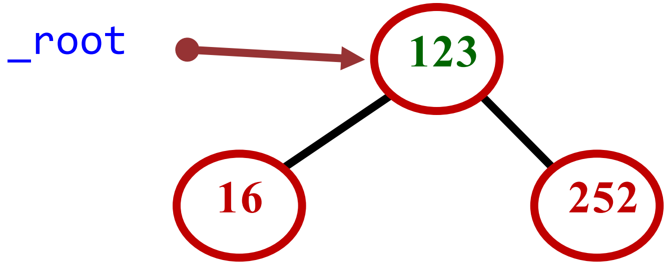
```
public Node<Type> insert(Type newData) {
    if (_data.compareTo(newData) > 0) { //newData should be in left subtree
        if(_left == null){ //left child is null – we've found the place to insert!
            _left = new Node(newData, this);
            return _left;
        } else{ //keep traversing down tree
            return _left.insert(newData);
        }
    } else { //newData should be in right subtree
        if(_right == null){//right child is null – we've found the place to insert!
            _right = new Node(newData, this);
            return _right;
        } else{ //keep traversing down tree
            return _right.insert(newData);
        }
    }
}
```

- Reference to the new Node is passed up the tree so it can be returned by the tree

Insertion Simulation (1/4)

- Insert: **224**
- First call insert in BST:

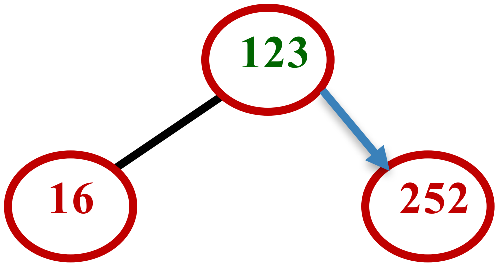
```
_root = _root.insert(newData);
```



Insertion Simulation (2/4)

- **123** says: "I am less than **224**. I'll let my right child deal with it.

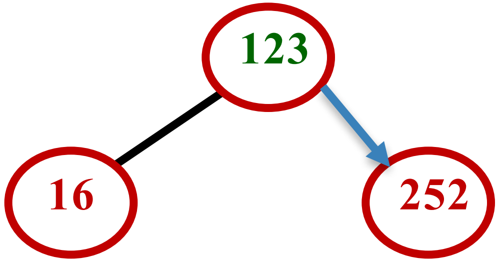
```
if (_data.compareTo(newData) > 0) {
        //code for inserting left elided
  } else {
        if(_right == null){
            //code for inserting with null
           //right child elided
        } else{
            return _right.insert(newData);
        }
}
```



Insertion Simulation (3/4)

- 252 says: “I am greater than 224. I’ll pass it on to my left child – but my left child is `null`!”

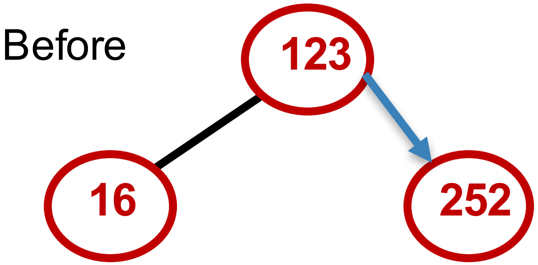
```
if (_data.compareTo(newData) > 0){
    if(_left == null){
        _left = new Node(newData, this);
        return _left;
    } else{
        //code for continuing traversal elided
    }
}
```

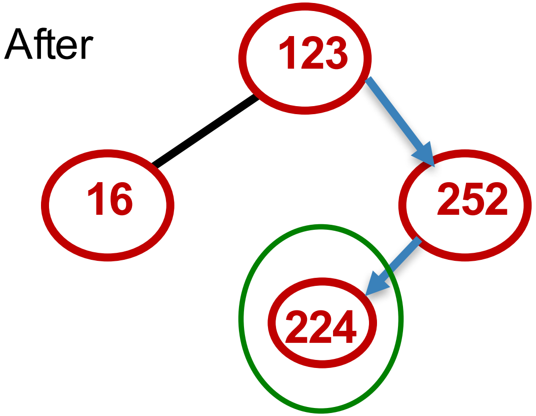


Insertion Simulation (4/4)

- 252 says: “You belong as my left child, 224. Let me make a node for you, make this new node your home, and set that node as my left child.”

```
_left = new Node(newData, this);
return _left;
```





Notes on Trees (1/2)

- Different insertion order of nodes results in different trees
  - if you insert a node referencing data value of 18 into empty tree, that node will become root
  - if you then insert a node referencing data value of 12, it will become left child of root
  - however, if you insert node referencing 12 into an empty tree, it will become root
  - then, if you insert one referencing 18, that node will become right child of root
  - even with same nodes, different insertion order makes different trees!
  - on average, for reasonably random (unsorted) arrival order, trees will look similar in depth so order doesn't really matter

Notes on Trees (2/2)

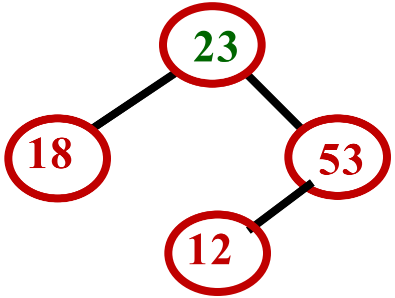
- When searching for a value, reaching another value that is greater than the one being searched for does not mean that the value being searched for is not present in tree (whereas it does in linked lists!)
    - it may well still be contained in left subtree of node of greater value that has just been encountered
    - thus, where you might have given up in linked lists, **you can't give up here until you reach a leaf** (but depth is roughly $\log_2 N$ which is much smaller than N/2!)

Clicker Question

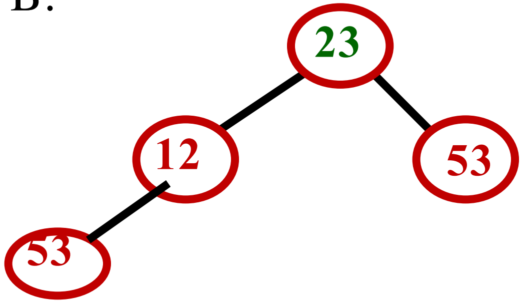
```
Node root = new Node(23, null)
root.insert(18)
root.insert(12)
root.insert(53)
```

Which tree does the code at the top right represent below?

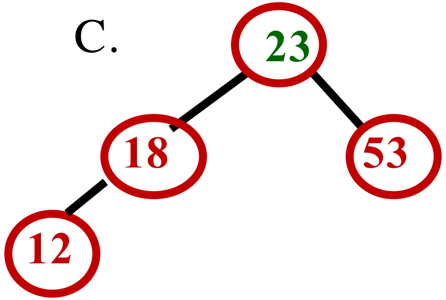
A.



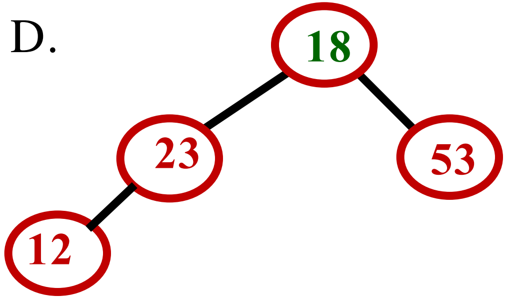
B.



C.



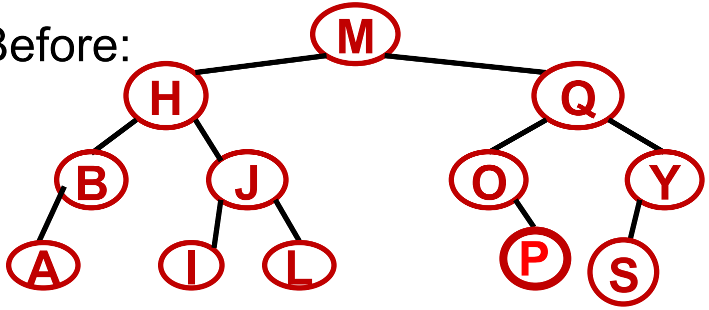
D.



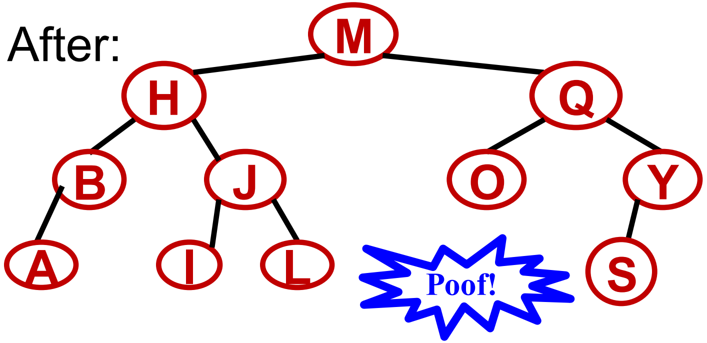
Remove: no child case

- Node to remove has no children(is a leaf)
  - just set the parent's reference to this Node to null – no more references means the Node is garbage collected!
- Example: Remove **P**
  - Set **O**'s right child to null, and **P** is gone!

Before:

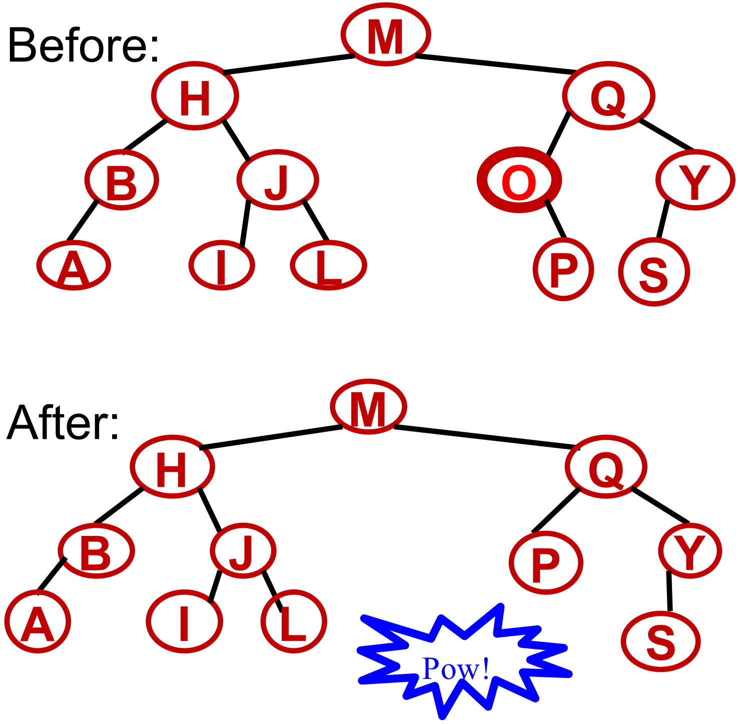


After:



Remove: one-child case

- Harder case: Node to delete has one child
  - replace Node child
- Example: Remove O
  - O has one child
  - Q replaces O by replacing its left child, previously O, with P



Remove: two-children case (1/3)

- Hard case: node to remove has two internal children
  - brute force: just flag node for removal, and rewrite tree at a later time -- bad idea, because now every operation requires checking that flag. Instead, do the work right away
  - this is tricky, because not immediately obvious which child should replace its parent
  - non-obvious solution: first swap the data in `Node` to be removed with data in a `Node` that doesn't have two children, then remove `Node` using one of simpler remove cases

Remove: two-children case (2/3)

- Use an auxiliary method, `swapData`
  - swaps data in node to be removed with the data in the right-most node in its left subtree
  - this child has a key value less than all `Node`s in the to-be removed `Node`'s right subtree, and greater than all other nodes in its left subtree
  - since it is a right-most `Node`, it has at most one child
  - this swap is temporary – we then remove the node in the right-most position using simpler remove

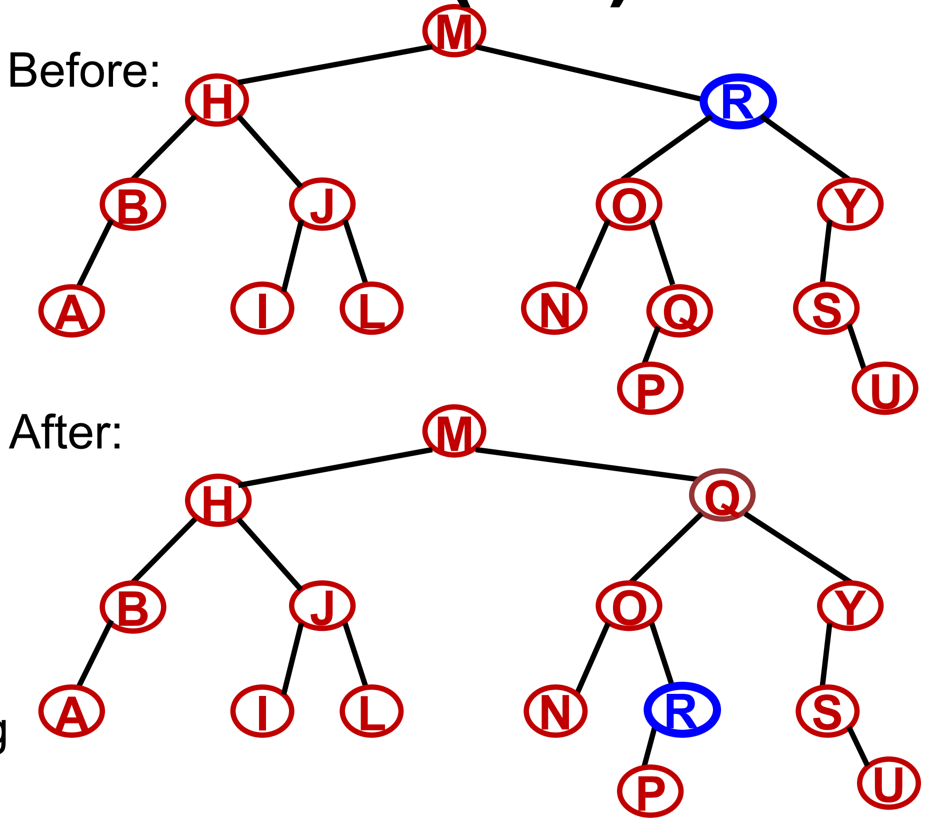
Remove: two-children case (3/3)

- Remove **R**
  - **R** has two children
  - swap **R** with the right-most Node in the left subtree, **Q**

    - Children in R's left subtree are smaller than **Q**

    - Children in **R**'s right subtree are larger than **Q**

    - **R** is in the wrong place but…
  - remove **R** (in its new position) using the one-child case



Remove: BST Code

- Starts as usual with delegating to root
- Need to first find the Node to remove, then we remove it
- Nodes are “smart,” so they can remove themselves
- $O(\log_2 N)$ because of searching

```
// in BinarySearchTree:
public void remove(Type dataToRemove) {
    Node<Type> toRemove = _root.search(dataToRemove);
    toRemove.remove();
}
```

Remove: Node Code (1/3)

- In the Node class, remove method allows Node to remove itself

```
public Node<Type> remove() {
    //Case 1 – Node to remove is a leaf node
    //Set its parent's reference that originally refers to this Node to null
    if(_left == null && _right == null){
        if(_parent.getLeft() == this){
            _parent.setLeft(null);
        }else{
            _parent.setRight(null);
        }
    }
    //Code for other cases on next slides…
}
```

Remove: Node Code (2/3)

```
public Node<Type> remove() {
    //code for case 1 elided
    //In a one-child case, we replace the _parent’s reference to Node with the Node’s child.
    } else if (_left != null && _right == null) { //case 2.1 – Node only has left child
        if (_parent.getLeft() == this){
            _parent.setLeft(_left);
        }else{
            _parent.setRight(_left);
        }
    } else if (_left == null && _right != null) { //case 2.2 – Node has only right child
        if (_parent.getLeft() == this){
            _parent.setLeft(_right);
        }else{
            _parent.setRight(_right);
        }
    } //Case 3 on next slide …
}
```

Remove: Node Code (3/3)

- Successor is guaranteed to have at most one child, so we remove with simpler remove case

```
public Node<Type> remove() {
    //code for case 1 (no children) elided
    //code for case 2 (one child) elided
    } else { //case 3 – both children
        Node<Type> toSwap = this.swapData();//swap data with successor
        toSwap.remove(); //now remove toSwap, which holds original Node’s data
        return toSwap; //return toSwap, since toSwap was data we removed
    }
    return this; //return this if we didn’t do any swapping since Node is removed
}
//swapData defined on next slide
```

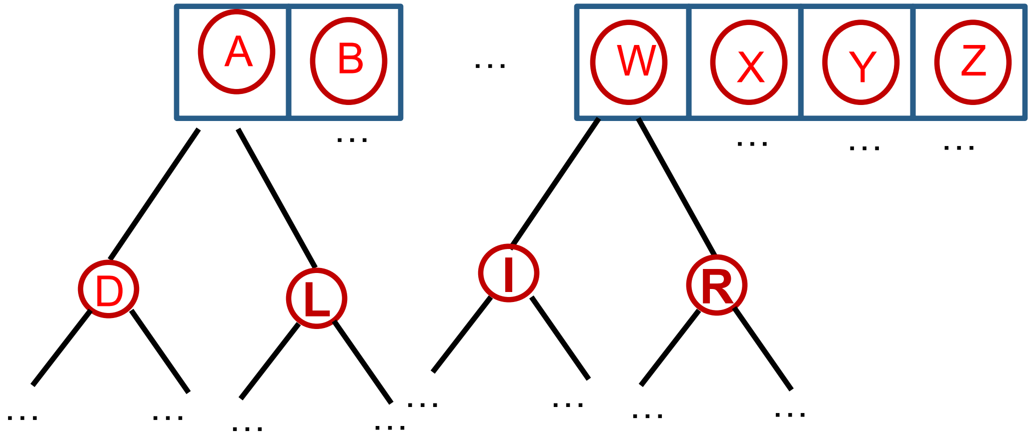
Remove: swapData code

- We find the right-most Node in left subtree, but we can also find the left-most Node in right subtree

```
public Node<Type> swapData(){
    Node<Type> curr = _left; //first get left child
    while(_left.getRight() ! = null){ //go right as far as possible
        curr = curr.getRight();
    }
    //swap data of this Node and successor
    Type tempData = _data;
    _data = curr.getData();
    curr.setData(tempData);
    return curr;
}
```

N-ary tree example

- Use the first character of last name as the start of a binary tree for all names with that initial character
    - 26-way division right away
- Disadvantages
    - Some trees will be very small, e.g. “q”, some will be much larger than average, e.g. “t”, “s”
    - Dividing by 26 doesn’t really get you that much (logn and logn/26 aren’t that different)



Tree Runtime

- Binary Search Tree has a search of $O(\log_2 n)$ → can we make it faster?
- Could make a ternary tree! (each node has at least 3 children)
  - $O(\log_3 n)$
- Or a 10-way tree with $O(\log_{10} n)$
- Let's try the runtime for a search with 1,000,000 nodes
  - $\text{Log}_{10} 1{,}000{,}000 = 6$
  - $\text{Log}_2 1{,}000{,}00 < 20$, so shallower but broader tree
- Analysis: the logs are not sufficiently different and the comparison (basically an n-way nested if-else-if) is far more time consuming, hence not worth it
- Furthermore, binary tree makes it easy to produce an ordered list (see slide 64)

Hash Tables vs. Trees

- You might be asking "why use trees when hash tables have O(1) insert and remove?"
  - Hash Tables and Trees are different data structures used for different kinds of problems
- If you're only concerned with finding exact values, a hash table will be faster
  - You know the exact key to search for
  - Ex. Find a student's Banner ID given their name
  - key is name and value is Banner ID
- If you're trying to solve a nearest neighbors problem, a BST will be faster
  - You do not know the exact key to search for
  - Ex. Find 4 people closest to a 95 in the class
  - key is grade and value is student name
- Can produce an already sorted list of data items by traversing the tree

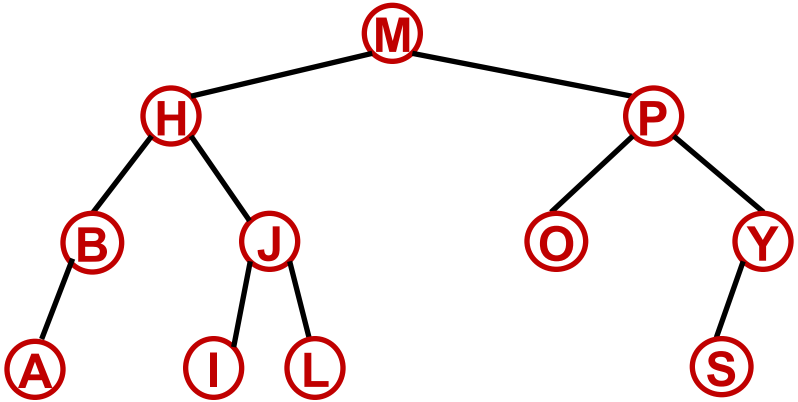
Traversing a Binary Tree

- We often want to access every Node in tree
  - so far, we have only searched for a single element
  - we can use a traversal algorithm to perform some arbitrary operation on every Node in tree
- Many ways to traverse Nodes in tree
  - order children are visited is important
  - three traversal types: inorder, preorder, postorder
- Exploit recursion!
  - subtree has same structure as tree

Inorder Traversal of BST

- Considered "in order" because `Node`s are visited in sorted order
- Traverse left subtree first, then visit self, then traverse right subtree
- Use recursion!

```
public void inOrder() {
    //Check for null children elided
     _left.inOrder();
     this.doSomething();
     _right.inOrder();
}
```

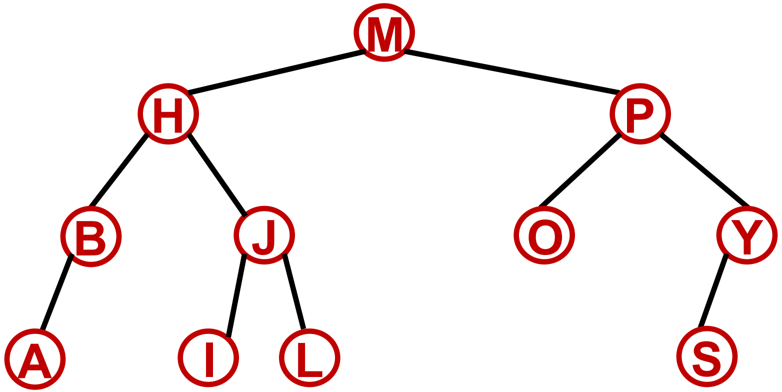


A B H I J L M O P S Y

Preorder Traversal of BST

- Preorder traversal
  - "Preorder" because self is visited before ("pre") visiting children
  - Again, use recursion!

```
public void preOrder() {
      //Check for null children elided
      this.doSomething();
      _left.preOrder();
      _right.preOrder();
}
```

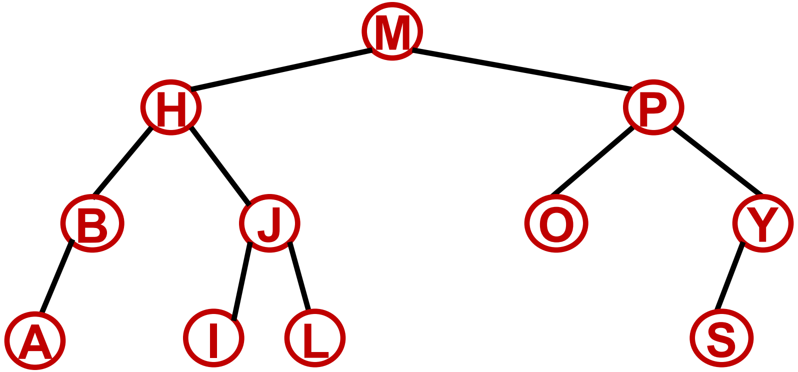


M H B A J I L P O Y S

Postorder Traversal of BST

- Postorder traversal
  - "Post-order" because self is visited after ("post-") visiting children
  - Again, use recursion!

```
public void postOrder() {
    //Check for null children elided
    _left.postOrder();
    _right.postOrder();
    this.doSomething();
}
```



A B I L J H O S Y P M

To learn more about the exciting world of trees, take CS16 (CSCI0160): Introduction to Algorithms and Data Structures!

Prefix, Infix, Postfix Notation for Arithmetic Expressions

- Infix, Prefix, and Postfix refer to where the operator goes relative to its operands
  - Infix: (fully parenthesized)

((1 * 2) + (3 * 4)) - ((5 - 6) + (7 / 8))

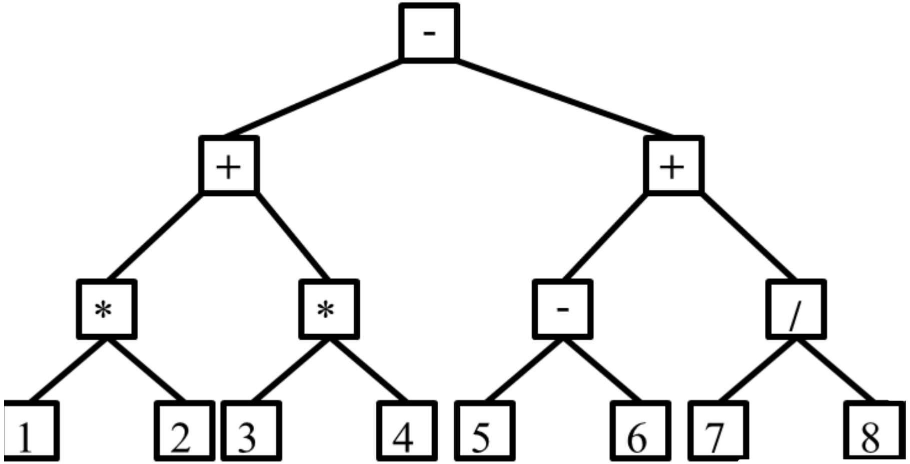
  - Prefix:

- + * 1 2 * 3 4 + - 5 6 / 7 8

  - Postfix:

1 2 * 3 4 * + 5 6 - 7 8 / + -

- Graphical representation for equation:



Using Prefix, Infix, Postfix Notation

- When you type an equation into a spreadsheet, you use Infix; when you type an equation into many Hewlett-Packard calculators, you use Postfix, also known as “Reverse Polish Notation,” or “RPN,” after its inventor Polish Logician Jan Lukasiewicz (1924)

- Easier to evaluate Postfix because it has no parenthesis and evaluates in a single left-to-right pass

- Use Dijkstra’s 2-stack shunting yard algorithm to convert from user-entered Infix to easy-to-handle Postfix – compile or interpret it on the fly

Dijkstra’s infix-to-postfix Algorithm (1/2)

2 stack algorithm for single-pass Infix to Postfix conversion, using operator precedence

(a + (b * (c ^ d)))   a b c d ^ * +

Use rule matrix to implement strategy

A) Push operands onto operand stack; push operators in precedence order onto the operator stack

B) When precedence order would be disturbed, pop operator stack until order is restored, evaluating each pair of operands popped from the operand stack and pushing the result back onto the operand stack.

Note that equal precedence displaces. At the end of the statement (marked by ; or CR) all operators are popped.

C) “(“ starts a new substack;

“)” pops until its matching “(“

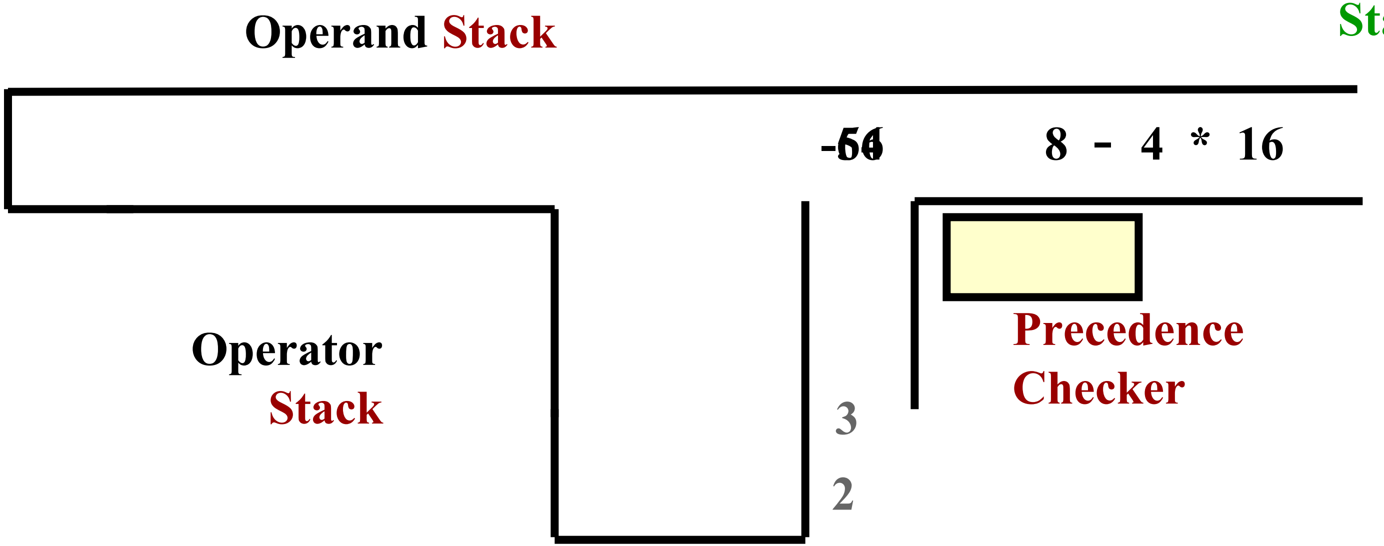
Incoming Operator

Top of Stack	(	^	*/	+-	)
(	A	A	A	A	C
^	A	B	B	B	C
*/	A	A	B	B	C
+-	A	A	A	B	C
e	A	A	A	A	E

Note: our Stack implementation doesn’t allow accessing the top-element without popping it; Java’s implementation has a peek method

Dijkstra's infix-to-postfix Algorithm (2/2)

(a + (b * (c ^ d)))   a b c d ^ * +



Top of Stack \ Incoming Operator	(	^	*/	+-	)
(	A	A	A	A	C
^	A	B	B	B	C
*/	A	A	B	B	C
+-	A	A	A	B	C
e	A	A	A	A	E

Challenge Questions Solutions

- **Q:** How would you print the elements of a Binary Search Tree in increasing order?
- **A:** You would traverse the BST **in-order**
- **Q:** How would you find the 'successor' (i.e., next greatest number) of a node in a Binary Search Tree?
- **A:** The pseudo-code for the solution is to find left-most node of right subtree since it is the smallest of the ones that are greater:

```
if node.hasRight()
        node=node.right()
        while(node.hasLeft())
                node=node.left()
return node
```

Announcements

- Lab 7 is due today at lab or Sunday 11/13 at TA Hours
- Lab 8 is due today at lab or Tuesday 11/15 at TA Hours
    - You know the exact key to search for
    - Ex. Find every person in the class that has a birthday on 05/08
    - Use a Hash Table where key is birthday, and value is CS15Student
- Data Structures and Algorithms discussion will happen next week
- Tetris deadlines
    - **Early**: 11/18, 10:00pm
    - **Ontime:** 11/20, 11:59pm
    - **Late:** 11/22, 11:59pm
    - If you missed your design discussion on Wednesday, there will be a make up design discussion **tomorrow at 3pm in CIT 368**

Announcements

- Next week's lectures are ***very very important***
    - Tuesday: Final Project demos
    - Thursday: Final Project help sessions
- There will be skits and super cool demos- come to class!