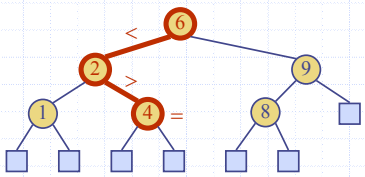
Binary Search Trees

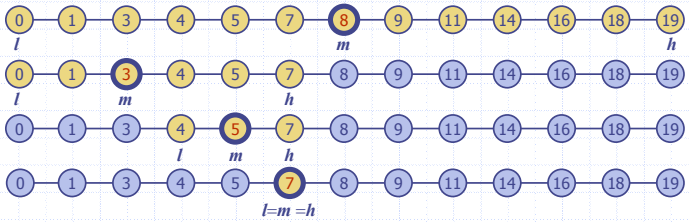


Ordered Dictionaries

- Keys are assumed to come from a total order.
- New operations:
  - first(): first entry in the dictionary ordering
  - last(): last entry in the dictionary ordering
  - successors(k): iterator of entries with keys greater than or equal to k; increasing order
  - predecessors(k): iterator of entries with keys less than or equal to k; decreasing order

Binary Search (§ 8.3.3)

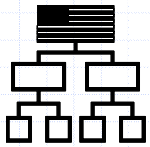
- Binary search can perform operation find(k) on a dictionary implemented by means of an array-based sequence, sorted by key
  - similar to the high-low game
  - at each step, the number of candidate items is halved
  - terminates after O(log n) steps
- Example: find(7)



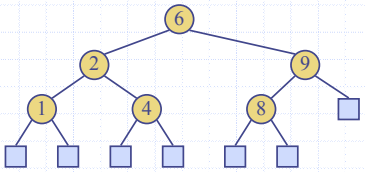
Search Tables

- A search table is a dictionary implemented by means of a sorted sequence
  - We store the items of the dictionary in an array-based sequence, sorted by key
  - We use an external comparator for the keys
- Performance:
  - find takes $O(\log n)$ time, using binary search
  - insert takes $O(n)$ time since in the worst case we have to shift $n/2$ items to make room for the new item
  - remove take $O(n)$ time since in the worst case we have to shift $n/2$ items to compact the items after the removal
- The lookup table is effective only for dictionaries of small size or for dictionaries on which searches are the most common operations, while insertions and removals are rarely performed (e.g., credit card authorizations)

Binary Search Trees (§ 9.1)



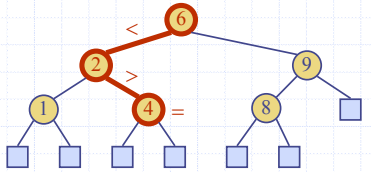
- A binary search tree is a binary tree storing keys (or key–value entries) at its internal nodes and satisfying the following property:
  - Let u, v, and w be three nodes such that u is in the left subtree of v and w is in the right subtree of v. We have $key(u) \leq key(v) \leq key(w)$
- External nodes do not store items
- An inorder traversal of a binary search trees visits the keys in increasing order



Search (§ 9.1.1)

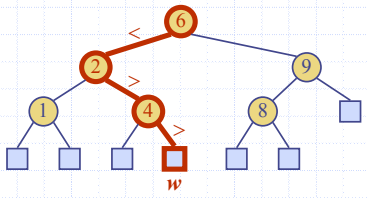
- To search for a key k, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of k with the key of the current node
- If we reach a leaf, the key is not found and we return nukk
- Example: find(4):
  - Call TreeSearch(4,root)

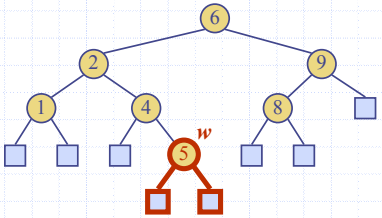
```
Algorithm TreeSearch(k, v)
  if T.isExternal (v)
    return v
  if k < key(v)
    return TreeSearch(k, T.left(v))
  else if k = key(v)
    return v
  else { k > key(v) }
    return TreeSearch(k, T.right(v))
```



Insertion

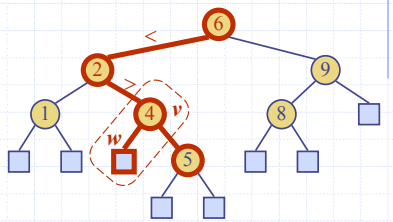
- To perform operation inser(k, o), we search for key k (using TreeSearch)
- Assume k is not already in the tree, and let let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert 5

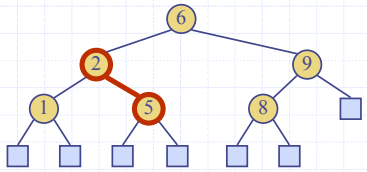




Deletion

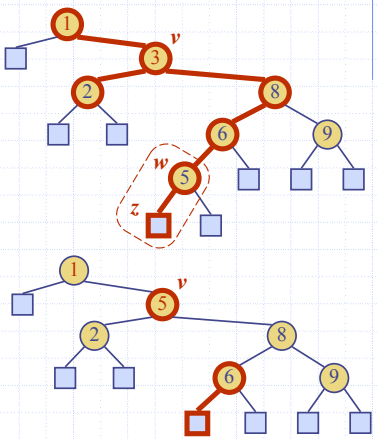
- To perform operation remove(k), we search for key k
- Assume key k is in the tree, and let let v be the node storing k
- If node v has a leaf child w, we remove v and w from the tree with operation removeExternal(w), which removes w and its parent
- Example: remove 4





Deletion (cont.)

- We consider the case where the key k to be removed is stored at a node v whose children are both internal
  - we find the internal node w that follows v in an inorder traversal
  - we copy $key(w)$ into node v
  - we remove node w and its left child z (which must be a leaf) by means of operation removeExternal(z)
- Example: remove 3



Performance

- Consider a dictionary with n items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods find, insert and remove take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case