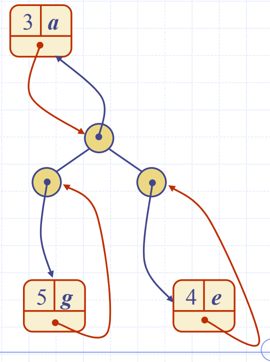
Adaptable Priority Queues



Recall the Entry and Priority Queue ADTs (§ 7.1)

- An **entry** stores a (key, value) pair within a data structure
- Methods of the entry ADT:
  - key(): returns the key associated with this entry
  - value(): returns the value paired with the key associated with this entry
- Priority Queue ADT:
  - insert(k, x) inserts an entry with key k and value x
  - removeMin() removes and returns the entry with smallest key
  - min() returns, but does not remove, an entry with smallest key
  - size(), isEmpty()

Motivating Example

- Suppose we have an online trading system where orders to purchase and sell a given stock are stored in two priority queues (one for sell orders and one for buy orders) as (p,s) entries:
  - The key, p, of an order is the price
  - The value, s, for an entry is the number of shares
  - A buy order (p,s) is executed when a sell order (p',s') with price p'≤p is added (the execution is complete if s'≥s)
  - A sell order (p,s) is executed when a buy order (p',s') with price p'≥p is added (the execution is complete if s'≥s)
- What if someone wishes to cancel their order before it executes?
- What if someone wishes to update the price or number of shares for their order?

Methods of the Adaptable Priority Queue ADT (§ 7.4)

- remove(*e*): Remove from *P* and return entry *e*.
- replaceKey(*e*,*k*): Replace with *k* and return the key of entry *e* of *P*; an error condition occurs if *k* is invalid (that is, *k* cannot becompared with other keys).
- replaceValue(*e*,*x*): Replace with *x* and return the value of entry *e* of *P*.

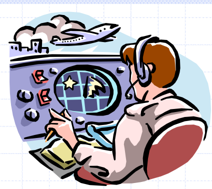
Example

Operation	*Output*	P
insert($5,A$)	e_1	$(5,A)$
insert($3,B$)	e_2	$(3,B),(5,A)$
insert($7,C$)	e_3	$(3,B),(5,A),(7,C)$
min()	e_2	$(3,B),(5,A),(7,C)$
key(e_2)	3	$(3,B),(5,A),(7,C)$
remove(e_1)	e_1	$(3,B),(7,C)$
replaceKey(e_2,9)	3	$(7,C),(9,B)$
replaceValue(e_3,D)	C	$(7,D),(9,B)$
remove(e_2)	e_2	$(7,D)$

Locating Entries

- In order to implement the operations remove(k), replaceKey(e), and replaceValue(k), we need fast ways of locating an entry e in a priority queue.
- We can always just search the entire data structure to find an entry e, but there are better ways for locating entries.

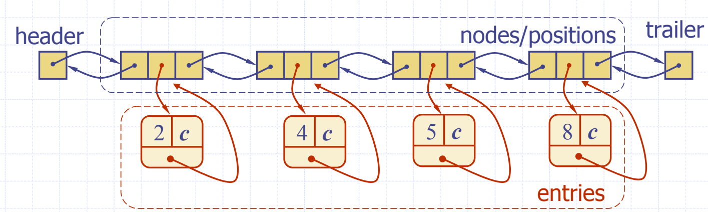
Location-Aware Entries



- A locator-aware entry identifies and tracks the location of its (key, value) object within a data structure
- Intuitive notion:
  - Coat claim check
  - Valet claim ticket
  - Reservation number
- Main idea:
  - Since entries are created and returned from the data structure itself, it can return location-aware entries, thereby making future updates easier

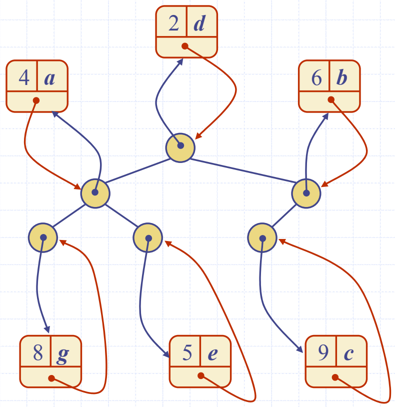
List Implementation

- A location-aware list entry is an object storing
  - key
  - value
  - position (or rank) of the item in the list
- In turn, the position (or array cell) stores the entry
- Back pointers (or ranks) are updated during swaps



Heap Implementation

- A location-aware heap entry is an object storing
  - key
  - value
  - position of the entry in the underlying heap
- In turn, each heap position stores an entry
- Back pointers are updated during entry swaps



Performance

- Using location-aware entries we can achieve the following running times (times better than those achievable without location-aware entries are highlighted in red):

Method	**Unsorted List**	**Sorted List**	**Heap**
size, isEmpty	$O(1)$	$O(1)$	$O(1)$
insert	$O(1)$	$O(n)$	$O(\log n)$
min	$O(n)$	$O(1)$	$O(1)$
removeMin	$O(n)$	$O(1)$	$O(\log n)$
remove	$\mathbf{O(1)}$	$\mathbf{O(1)}$	$\mathbf{O(\log n)}$
replaceKey	$\mathbf{O(1)}$	$O(n)$	$\mathbf{O(\log n)}$
replaceValue	$\mathbf{O(1)}$	$\mathbf{O(1)}$	$\mathbf{O(1)}$